VOL. XLIX. JANUARY, 1895.

Established by **BENJAMIN SILLIMAN** in 1818.

THE

AMERICAN

JOURNAL OF SCIENCE.

EDITORS

JAMES D. AND EDWARD S. DANA.

ASSOCIATE EDITORS

PROFESSORS GEO. L. GOODALE, JOHN TROWBRIDGE AND HENRY P. BOWDITCH, OF CAMBRIDGE.

PROFESSORS H. A. NEWTON, O. C. MARSH, A. E. VERRILL AND H. S. WILLIAMS, OF NEW HAVEN,

PROFESSOR GEORGE F. BARKER, OF PHILADELPHIA.

THIRD SERIES.

VOL. XLIX—[WHOLE NUMBER, CXLIX.]

No. 289.—JANUARY, 1895.

NEW HAVEN, CONN.: J. D. & E. S. DANA.

1895.

TUTTLE, MOREHOUSE & TAYLOR, PRINTERS, 125 TEMPLE STREET.

Published monthly. Six dollars per year (postage prepaid). $6.40 to foreign subscribers of countries in the Postal Union. Remittances should be made either by money orders, registered letters, or bank checks.

MINERALS.

Recently collected by Dr. Foote, in Arkansas.

Quartz, From Hot Springs.

Rare modifications! Beautiful inclusions!

Besides the clear crystals that have been famous for half a century, this locality has recently produced most beautiful inclusions of *Chlorite, Manganese oxide,* crystals of *Albite, Pyrophyllite,* etc.

In many cases these inclusions are deposited along the lines of the Quartz crystallization, giving the effect of "phantom crystals." These "phantoms" are particularly striking when the effect is due to the green chloritic "moss," a number of them having been cut as gems.

A few crystals showing the right and left-handed modifications, and the twinning of the two, (Dana, '92 ed., p. 184, figs. 5, 7, 8, 9, and 10); remarkably flattened, curved and other distorted forms were obtained.

The beauty and variety of these can be fully appreciated only by those who saw the great collection of Dr. G. W. Lawrence in the Arkansas building at the Centennial.

For forty years the Doctor had collected while resident physician at Hot Springs, and at his death, three years ago, his collection was unequalled in Arkansas minerals. It had not been unpacked until purchased last month by Dr. Foote, who at the same time got the pick of several other fine old private collections, and had men blasting for him at Magnet Cove, during the several weeks that he spent in and around Hot Springs.

These and other newer collections of Hot Springs Quartz that have just reached us, aggregate nearly seven thousand pounds, and while the vast majority of the specimens are beautiful clusters, and simple crystals, of the type form, the lot embraces also many varieties that will probably never be duplicated.

Quartz, *clear single crystals,* 5c. to $5.00; magnificent groups for open cases, $5.00 to $35.00; smaller, but equally attractive, 2 to 8 inches diameter, 5c. to $7.50. *Inclusions* of different minerals, showing the "*phantom*" effect, 25c. to $15.00. *Containing moving bubbles,* 50c. to $5.00. *Inclusions* of Pyrophyllite (unaltered blue-green) and Albite crystals, 25c. to $5.00. *Modified,* bent, flattened, twisted, showing rare planes, 25c. to $10.00. *Cavernous, Distorted, Interpenetrations,* etc., 5c. to $5.00.

From Magnet Cove.

Monticellite, *Vesuvianite* in large single crystals and groups, *Leucite* (a new discovery), *Pseudoleucite, Brookite, Rutile* rosettes, *Eudialyte, Schorlomite, Protovermiculite, Lodestone, Elæolite, Aegirite, Ozarkite, Melanite, Dysanalyte* in gangue, etc., etc.

From Montgomery Co.

Wavellite in beautiful bright green radiations, and occasionally in balls of terminated crystals, 5c. to $5.00. *Variscite,* finest color, 5c. to $1.50.

Utah Selenite. In *small crystals,* and crystals and cleavages showing *moving bubbles,* 25c. to $5.00. A few extraordinary specimens at higher prices; one limpid cleavage two and one-half feet long, with bubbles moving nearly the entire length, $50.00.

Diaspore! *The best six specimens in the world.* Clear good sized violet colored crystals in exquisite groups. *Crystallized Margarite and Corundophilite* all from Chester, Mass.

Realgar from Greece, *Kylindrite* from Bolivia, and many others.

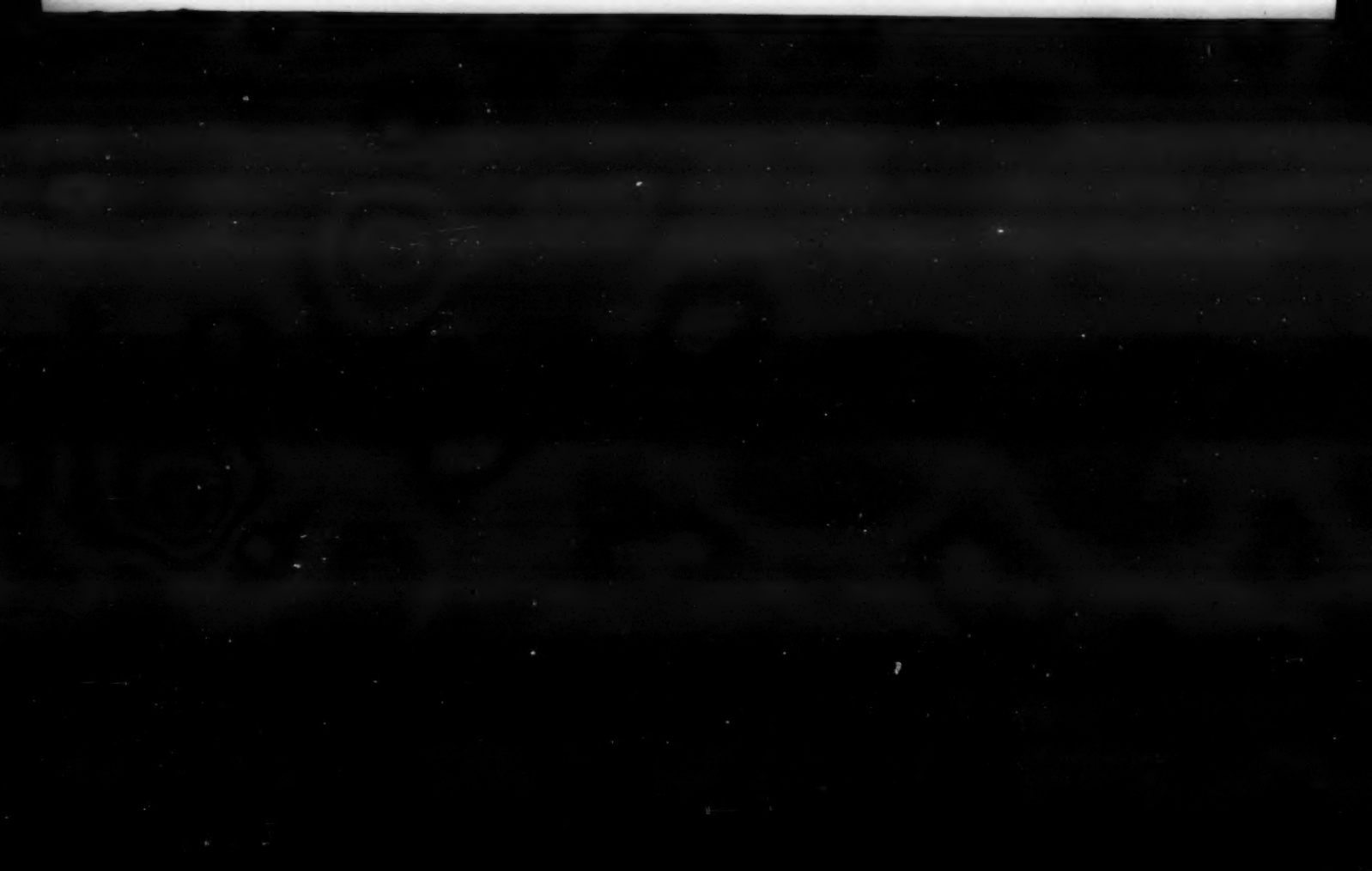
Price Lists and Circulars Free on Application.

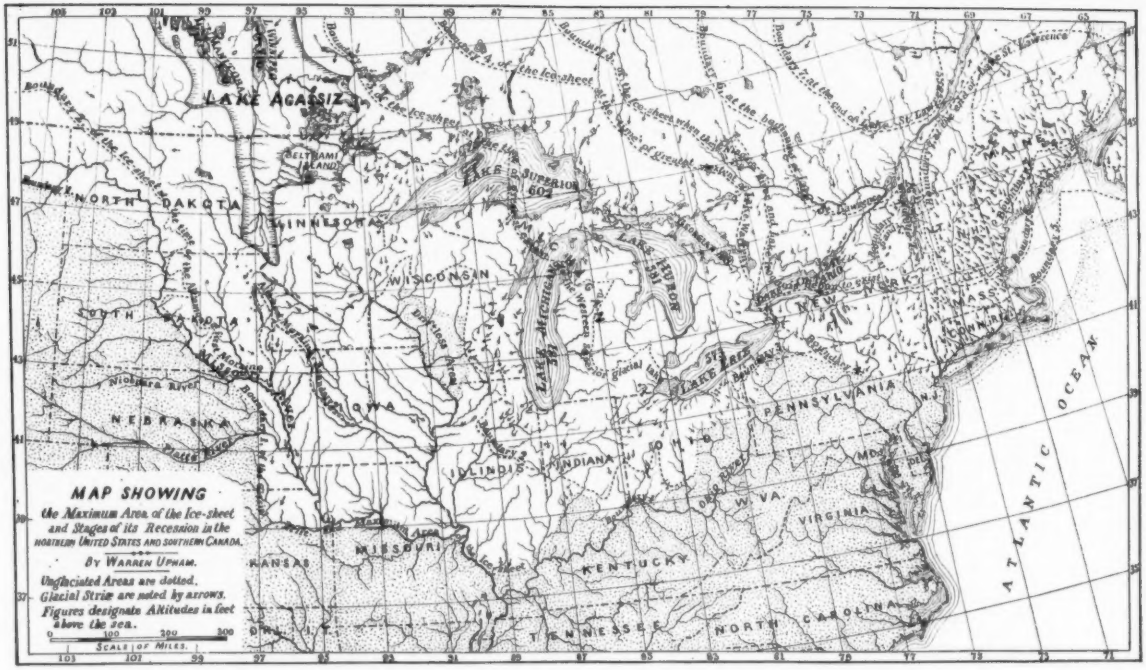
Catalogue of Minerals, 128 pp. with plates and illustrations, heavy paper, 10c. Cloth, 25c.; Boards, 20c.; Half Morocco, 50c.

MINERALS, SCIENTIFIC AND MEDICAL BOOKS,

DR. A. E. FOOTE, 1224-26-28 North 41st Street,

Philadelphia, Pa., U. S. A.





STAGES OF RECESSION OF THE NORTH AMERICAN ICE-SHEET.

THE

AMERICAN JOURNAL OF SCIENCE

[THIRD SERIES.]

ART. I.—*Late Glacial or Champlain Subsidence and Reëlevation of the St. Lawrence river basin;* by WARREN UPHAM. (With Plate I.)

THE accompanying map (Plate I) shows the maximum area covered by the ice-sheet in the St. Lawrence basin and adjoining portions of the United States and southern Canada, with approximate outlines of the glacial boundary at successive stages of its retreat. The Champlain epoch or part of the Pleistocene period including these stages of glacial recession was begun and ended, respectively, by downward and upward epeirogenic movements. It comprised the time of departure of the ice-sheet, with many small and large glacial lakes temporarily formed by its receding barrier, and with marine submergence to hundreds of feet above the present shore lines. The Late Glacial subsidence appears to have been principally completed before the retreat of the ice and deposition of the Champlain lacustrine and marine beds; but the following uplift was in progress, advancing as fast as the ice receded, from the beginning to the end of Champlain time.* Indeed, considerable parts of the glaciated areas of North America and Scandinavia are still undergoing small and slow oscillatory movements, not having yet, during the short Postglacial period, fully reached isostatic repose.

*For a discussion of the part of this movement reëlevating the upper Mississippi region, the area of the glacial Lake Agassiz in the basin of the Red river of the North and Lake Winnipeg, and the country surrounding Hudson Bay, see the Journal of Geology, vol. ii, pp. 383–395, May–June, 1894. The dynamic causes of epeirogenic movements, and their relations to the Glacial period as the probable causes of both its beginning and end, are partly considered in that paper, but more fully in an appendix of Wright's Ice Age in North America, 1889, pp. 573–595, this Journal, III, vol. xlvi, pp. 114–121, Aug., 1893, and the Geol. Magazine, IV, vol. i, pp. 340–349, Aug., 1894.

Evidence from the Beaches of the Glacial Lakes in the St. Lawrence basin.

Well marked old channels of outflow are found extending southward, at the levels of the deserted beaches, from Lake Agassiz and from the glacial lakes which are now represented by the diminished, but still large, modern lakes Superior, Michigan, Huron, Erie, Ontario, and Champlain. The outlets prove that the great Pleistocene water bodies which occupied these basins were lakes, not gulfs or arms of the sea; and the differential uplifts of the basins, increasing toward the central part of the area of the continental ice-sheet, show that no land barriers, but the ice itself in its retreat, held in these lakes on their northward sides.

The basin of the St. Lawrence during the glacial recession held successively, and in part contemporaneously, no less than eight important glacial lakes, distinguished by their different areas, heights, and places of outlet. They are named the Western Superior and Western Erie glacial lakes; Lake Warren, the most extensive, into which the two foregoing were merged; Lake Algonquin, the successor of Lake Warren in the basins of Lakes Huron, Michigan, and Superior; Lake Lundy, the glacial representative of Lake Erie; Lake Iroquois, in the basin of Lake Ontario; Lake Hudson-Champlain; and Lake St. Lawrence, into which the two last named became merged. The glacial Lake St. Lawrence, which is the only one of the series hitherto unnamed, extended over the Ottawa, Champlain, and St. Lawrence valleys previous to the melting away of the ice barrier, remaining latest in the vicinity of Quebec, by which event the sea, at a lower level than the former lake, was admitted to these valleys.

*The Western Superior glacial lake.**—In the west part of the basin of Lake Superior the receding ice-sheet held a lake which outflowed southward through northwestern Wisconsin, across the present watershed between the Bois Brulé and St. Croix rivers. The highest shore line of this lake at Duluth is 535 feet above Lake Superior (which has a mean level 602 feet above the sea); on Mt. Josephine, about 130 miles northeast from Duluth, its height, according to leveling by Dr. A. C. Lawson,† is 607 feet; and at L'Anse and Marquette, Mich., 175 and 225 miles east of Duluth, it is found by Mr. F. B. Taylor‡ about 590 feet above the lake. The northeastward

* Proc. A. A. A. S., vol. xxxii, for 1883, p. 230. Geol. and Nat. Hist. Survey of Minnesota, Final Report, vol. ii, 1888, p. 642; Twenty-second Ann. Rep. for 1893, pp. 54–66 (first use of this name). Bulletin Geol. Soc. Am., vol. ii, 1891, p. 258. Am. Geologist, vol. xi, p. 357, May, 1893; and vol. xiv, p. 63, July, 1894.

† Minnesota Geol. Survey, Twentieth Ann. Rep. for 1891, pp. 181–289, with map and profiles.

‡ Am. Geologist, vol. xiii, pp. 316–327 and 365–383, with maps, May and June, 1894.

uplift averages seven inches per mile; and the eastward ascent is approximately three inches per mile. The latest and lowest of the Western Superior lake beaches observed at Duluth, occupied by the "boulevard" or pleasure driveway, 475 feet above the lake, on the bluffs back of the city, appears to have an ascent of only about 35 feet in the distance to Mt. Josephine, showing that the uplift of the land was quite rapidly in progress while the ice-front still maintained the lake at the St. Croix outlet. Not long after the glacial retreat passed eastward beyond Mt. Josephine and Marquette, this lake was lowered and merged with Lake Warren across the lowlands of the northern peninsula of Michigan. The vertical interval between the final stage of the Western Superior Lake and the level of Lake Warren shown by its earliest beach at Duluth was about 60 feet. Thenceforward the outlet of Lake Warren past Chicago carried away the drainage from the glacial melting and rainfall of the Superior basin.

*The Western Erie glacial lake.**—Outflowing from the southwestern end of the Lake Erie basin by a large abandoned watercourse, which reaches from Ft. Wayne, Ind., where the St. Joseph's and St. Mary's rivers unite to form the Maumee, across the present watershed to the Wabash river, this glacial lake formed two distinct beaches, named by N. H. Winchell the Van Wert and Leipsic ridges, separated by a vertical interval of 15 to 20 or 25 feet. The upper or Van Wert beach, with its crest varying in altitude from 200 to 220 feet above Lake Erie (whose mean height is 573 feet above the sea), extends about 75 miles east to Findlay, Ohio, and nearly an equal distance northeast past Bryan, Ohio, to the vicinity of Adrian, Mich., if not farther. At Findlay the lake while forming this beach, as Winchell and Leverett have shown, was bounded on the north by the ice-sheet then forming the Blanchard moraine. The second or Leipsic beach of the Western Erie Lake, ranging in height from 190 to 210 feet, runs from Ft. Wayne eastward 175 miles to its termination, as described by Leverett, at the line of a later moraine close southwest of

* G. K. Gilbert, this Journal, III, vol. i, pp. 339–345, with map, May, 1871; Geology of Ohio, vol. i, 1873, pp. 540–556, with two maps. N. H. Winchell, Proc. A. A. A. S., vol. xxi, for 1872, pp. 171–179; Geology of Ohio, vol. ii, 1874, pp. 56, 431–433, etc. J. S. Newberry, Geology of Ohio, vol. ii pp. 46–65, with three maps and numerous sections. E. W. Claypole, "The Lake Age in Ohio," Trans. Geol. Soc. Edinburgh, 1887, p. 42, with four maps. G. F. Wright, The Ice Age in North America, 1889, chapter xv (with reproduction of Prof. Claypole's maps, that of Lake Erie–Ontario being on p. 355). J. W. Spencer, this Journal, III, vol. xli, p. 208, with map, March, 1891; Bulletin, Geol. Soc. Am., vol. ii, 1891, pp. 465–476, with map. Frank Leverett, this Journal, III, vol. xliii, pp. 281–297, with map, April, 1892. Warren Upham, Bulletin, Geol. Soc. Am., vol. ii, p. 259; Minnesota Geol. Survey, Twenty-second Ann. Rep. for 1893, p. 62 (first use of the name *Western Erie Glacial Lake*).

Cleveland. Northeast and north from the old outlet the Leipsic beach reaches about 165 miles, past Adrian and Ypsilanti to Imlay, Mich., being nearly level to Ypsilanti, but thence in the 60 miles onward to Imlay having a rise of about 65 feet, to an altitude 849 feet above the sea. With the recession of the ice-sheet and the extension of this lake to Imlay, a lower outlet was opened over the watershed between the Shiawassee and Grand rivers in Michigan, 729 feet above the sea or 148 feet above Lakes Huron and Michigan, where the Western Erie glacial lake became confluent with Lake Warren and was thus reduced about 30 feet, falling from the Leipsic or lower Western Erie beach to the Belmore or earliest beach of Lake Warren in the Erie basin.

Upon a large area, extending from Ft. Wayne east to Cleveland and northward to Ypsilanti and Detroit, the attitude or general slopes and levels of the land have remained unchanged since the departure of the ice-sheet, for these earliest beaches and the lower beaches of Lake Warren in the same area are still nearly horizontal. The whole country there, however, has been uplifted, without tilting, about 110 feet, after the end of the separate existence of the Western Erie lake, for this is the height of the Belmore beach around the west end of Lake Erie above the highest and earliest beach of Lake Warren at Chicago. A greater and differential uplift, with rapid tilting of northward ascent, was taking place north and northeast of Detroit during the Belmore and lower stages of Lake Warren, simultaneous with the uniform elevation of the Western Erie glacial lake area. Further we learn that about half of the uplift of 110 feet for this region had occurred before the beginning of Lake Algonquin and the date of the Algonquin beach, since that beach has a height of 602 feet near the south end of Lake Huron, being 60 feet higher than the correlative sublacustrine terrace plane beneath the surface of Lake Michigan near Chicago, which marks the old Algonquin shore there.

*Lake Warren.**—Like the Western Superior and Western

* J. W. Spencer, Science, vol. xi, p. 49, Jan. 27, 1888 (proposing this name in honor of Gen. G. K. Warren); Proc. A. A. A. S., vol. xxxvii, for 1888, pp. 197–199; Trans. Roy. Soc. of Canada, vol. vii, for 1889, sec. iv, p. 122; this Journal, III, vol. xli, pp. 201–211, with map, March, 1891; Bulletin, Geol. Soc. Am., vol. ii, pp. 465–476, with map, April, 1891; "A Review of the History of the Great Lakes," Am. Geologist, vol. xiv, pp. 289–301, Nov., 1894 (containing citations of many additional papers by Prof. Spencer and others). G. K. Gilbert, "Changes of Level of the Great Lakes," in The Forum, vol. v, pp. 417–428, June, 1888; "History of the Niagara River," in Sixth Annual Report of the Commissioners of the State Reservation at Niagara, for 1889, pp. 61–84, with eight plates (also in the Smithsonian An. Rep. for 1890, pp. 231–257); Geology of Ohio, vols. i and ii. Frank Leverett, paper before cited; "Raised Beaches of Lake Michigan," Trans. Wisconsin Academy of Sciences, Arts, and Letters, vol. vii, pp. 177–192 (read Dec. 30, 1887). A. C. Lawson, "Sketch of the Coastal Topography of the North Side of Lake Superior, with Special Reference to the

Erie glacial lakes, the far more extensive Lake Warren at the beginning of its existence occupied only the southern end of the basin of Lake Michigan. It grew northward as the ice-sheet retired, and in due time it received these two lakes to itself, expanding thus into the basins of Lakes Superior, Huron, and Erie. The maximum development of Lake Warren stretched from Thomson, Minn., above and west of Duluth, eastward to Lake Nipissing, a distance of nearly 600 miles; and from Chicago, where it outflowed to the Des Plaines, Illinois, and Mississippi rivers, it extended eastward in its highest stages across the southern peninsula of Michigan, and later by way of the strait of Mackinaw and over Lakes Huron, St. Clair, and Erie, to the west end of the Lake Ontario basin and to Crittenden in southwestern New York. This area exceeded 100,000 square miles, being nearly equal to that of the glacial Lake Agassiz. The Belmore and Nelson beaches, the two highest formed by Lake Warren in the basins of Lakes Erie, Huron, and Superior, called by Spencer the Ridgeway beach (a later name than N. H. Winchell's "Belmore ridge") in their united course about the west half of Lake Erie, show that, since the fullest expansion of this great glacial lake, the whole basin of Lake Superior and the country eastward to Lake Nipissing have been uplifted 400 to 550 or 600 feet, in comparison with Chicago and the southern part of the Lake Michigan basin, while the uplift at Cleveland has been about 115 feet, and at Crittenden, N. Y., not less than 260 feet (more probably about 300 feet).

In the vicinity of Chicago, Lake Warren formed three beaches, belonging to lake levels successively about 45 to 50 feet, 15 feet, and 30 feet above Lake Michigan. That the

Abandoned Strands of Lake Warren," Minnesota Geol. Survey, Twentieth An. Rep. for 1891. pp. 181–289, with map, profiles, and figures from photographs. F. B. Taylor, this Journal, III, vol. xliii, pp. 210–218, March, 1892 (Mackinac island); Bulletin Geol. Soc. Am., vol. v, pp. 620–626, with maps, April, 1894 (Lake Nipissing); Am. Geologist, vol. xiii, pp. 316–327 (Green bay) and 365–383 (south coast of Lake Superior), with maps, May and June, 1894; id., vol. xiv, pp. 273–289 (east of Georgian bay), with map, Nov., 1894. The highest beach on Mackinac island, which Mr. Taylor calls the "Algonquin beach," seems to be correlative with his Nelson and higher beaches in the vicinity of Lake Nipissing, regarded in this paper as marking the early high stages of Lake Warren. C. Whittlesey, Smithsonian Contributions, vol. xv, 1864, pp. 17–22. E. Andrews, "The North American Lakes considered as Chronometers of Postglacial Time," Trans. Chicago Academy of Sciences, vol. ii. Nearly all the edition of this important paper was consumed in the Chicago fire of 1871. It is quite fully reproduced by James C. Southall, in "The Recent Origin of Man," 1875, chapter xxxiii (pp. 495–506, with sections); and in "The Epoch of the Mammoth and the Apparition of Man upon the Earth," 1878, chapter xxii (pp. 348–367, with sections). N. H. Winchell, J. S. Newberry, E. W. Claypole, and G. F. Wright, as before cited. Geol. Survey of Canada, Report of Progress to 1863, pp. 912, 913. Warren Upham, Bulletin Geol. Soc. Am., vol. ii, pp. 258–265; vol. iii, pp. 484–487. Geology of Minnesota, Twenty-second An. Rep. for 1893, as before cited. Am. Geologist, vol. xiv, pp. 62–65, July, 1894.

beach at 30 feet was formed after that at 15 feet is shown by the occurrence in some places of a peat deposit, described by Andrews and Leverett, which passes underneath the 30 feet beach and is continuous from its upper side down to the lower beach. The peat marks a land surface over which the lake rose to form the middle or third beach, after having stood at the lower or second beach for some time. Still later, however, it probably again stood at the lower level, corresponding to the present watershed in the abandoned outlet. This old channel of outflow, at its summit, as I am informed by Mr. Ossian Guthrie from the canal survey, is now 11 feet above the mean level of Lake Michigan, but the surface there is postglacial silt; at another point, where the channel bed consists of till, and at a third place where the bed is rock, its height in each case is only eight feet above the present lake, or 590 feet above the sea. The mouth of Lake Warren appears to have been at first near Lemont, on the Des Plaines river about 25 miles from the lake, where the river valley was obstructed by drift which suffered erosion, allowing the mouth of the lake to be transferred gradually upstream, at the same time being lowered, to its final position ten miles from the lake shore in Chicago. Epeirogenic movements, between the times of formation of the second and third beaches, slightly lifted the outlet and adjacent portion of the course of the Des Plaines river, as compared with the southern and southwestern part of the Lake Michigan basin, causing the old lake to extend a little farther on that side than before. Toward the north and east, however, this change was doubtless more than counteracted by the rapid differential rise of the land.

Fresh-water shells are found abundant in the 15 feet beach at Evanston and elsewhere southward through Chicago. All the species obtained, representing ten or more genera, are still living in this region. Wood of oak and cedar, and the thigh bone of a deer, have been also found in the same beach at Evanston.*

For the distance of about 185 miles from Chicago north to the south end of Green bay, the highest shore of Lake Warren appears to be now nearly level, for Mr. Taylor finds evidence of submergence only to a height of some 20 feet above that part of Green bay and the neighboring lake shore. Thence northward, however, the beach rises about 1·4 feet per mile for 110 miles to Cook's Hill, near the north end of this bay; in 60 miles from that latitude north to Houghton, it has an ascent of 260 feet, or $4\frac{1}{3}$ feet per mile; but in about 90

* H. M. Bannister, Geology of Illinois, vol. iii, 1868, pp. 241, 242. F. Leverett, "Raised Beaches of Lake Michigan," before cited, p. 189.

miles onward, across Lake Superior to Kaministiquia, where the shore is 455 feet above that lake, the rate of northward ascent is reduced to only a half of a foot per mile.

Along a west to east course, the Nelson beach (named by Taylor in the vicinity of North Bay, Lake Nipissing, probably not distinct from the Belmore beach in Ohio and northward to Mackinac island) is 385 feet above Lake Superior at Duluth; 410 feet at Houghton, having an eastward ascent of 25 feet in 150 miles; 414 feet at the Sault Ste. Marie, running level for 200 miles east from Houghton; and about 538 feet at the north side of Lake Nipissing, or 497 feet above that lake, and 1,140 feet above the sea. In the distance of 220 miles from Sault Ste. Marie to Lake Nipissing this beach now shows an ascent of 126 feet, or about seven inches per mile. These figures, with the preceding from Houghton to the north side of Lake Superior, justify to a remarkable degree Dr. Lawson's opinion that the ancient shore lines of Lake Warren in the Superior basin remain parallel with the water level of to-day. As compared with Chicago, the country enclosing Lake Superior has been uplifted 400 to 450 feet; and the greater part of the differential elevation, expressed by tilting, took place upon the west to east belt of the northern peninsula of Michigan.

Three beaches of Lake Warren are mapped by Spencer and named the Ridgeway, Arkona, and Forest beaches in Ohio, northwestern Michigan, and the province of Ontario north of Lake Erie. These probably represent the three noted at Chicago and about the south part of Lake Michigan. Farther north the number of distinct shore lines is much increased. In and near Duluth I find eight beaches referable to Lake Warren, the lowest being 50 feet above Lake Superior. On northern portions of the Lake Superior coast several of these seem, as shown by Lawson's observations with leveling, to be each represented by two or more shores, separated by vertical intervals of 10 feet or more. Most of the northern beaches, it should be remarked, are very feebly developed, even in the most favorable situations for their formation, and are not discernible along the far greater part of the lake borders. During all the time of differential uplifting of the Lake Warren basin and sinking of the water surface, whenever the diminishing lacustrine area was nearly unchanged for a few years or longer, the erosion and deposition effected by the great waves of storms, and the tribute of streams forming deltas, recorded these shore lines.*

* Prof. Spencer, in his latest paper ("A Review of the History of the Great Lakes," Am. Geologist, vol. xiv, pp. 298, 301, Nov., 1894), supposes that an outflow from Lakes Superior, Huron, Michigan, and Erie, passed by the way of

*Lake Algonquin.**—When the glacial melting and retreat at length permitted an outflow from the St. Lawrence basin over a lower pass, which was through central New York to the Mohawk and Hudson, the water surface of the basins of Lakes Michigan, Huron, and Superior, fell only some 50 or 75 feet, from the latest and lowest stage of Lake Warren to its short-lived successor, Lake Algonquin. This lake appears to have been ice-dammed only at low places on its east end, as at or near the heads of the Trent and Mattawa rivers, lying respectively east of Lakes Simcoe and Nipissing, where otherwise its waters must have been somewhat further lowered to outflow by these passes. Careful study and comparison of the work of Spencer in tracing the Algonquin beach about the southern part of Lake Huron and Georgian bay, and of Taylor in exploration of his "Nipissing beach" from Duluth east along the south coast of Lake Superior and the north side of Lake Huron and Georgian bay to Lake Nipissing, convince me that these beaches were of contemporaneous formation, marking respectively the southern and northern shores of Lake Algonquin, and therefore both to be known by the name Algonquin beach of Spencer, according to the law of priority. The earliest and principal stage of Lake Algonquin is shown by these beaches to have coincided closely in area with Lakes Michigan and Superior, but to have been considerably more extensive eastward than the present Lake Huron and Georgian bay. It held a level which now by subsequent differential epeirogenic movements is left probably wholly below the level of Lake Michigan by a vertical amount ranging from almost nothing to about 40 feet. Its shores were nearly coincident with the western shore of Lake Huron, but eastward they are now elevated mostly 150 to 200 feet above that lake and Georgian bay; and in the Lake Superior basin they vary from about 50 feet above Lake Superior at its mouth, and along its northeastern and northern shores, to 25 feet at Houghton, and to a few feet or none at Duluth.

The Algonquin beach at the south end of Lake Huron coincides very closely with the land surface there and with the

Chicago to the Des Plaines and Mississippi rivers so lately as about 1,500 years ago, when the Niagara river had cut back its gorge to the Johnson ridge, about a mile north of the present site of the falls. This would have formed a beach 10 to 15 feet above Lakes Michigan and Huron, and about 20 to 25 feet above Lake Erie, around all their shores; and the absence of such a modern and still horizontal shore line, slightly higher than the present lake levels, upon all this large area, forbids an acceptance of this hypothesis.

* J. W. Spencer, "Deformation of the Algonquin Beach, and Birth of Lake Huron," this Journal, III, vol. xli, pp. 12–21, with map, Jan., 1891; and other papers before cited. G. K. Gilbert, F. B. Taylor, and Warren Upham, as before cited for Lake Warren. G. F. Wright, Bulletin Geol. Soc. Am., vol. iv, pp. 423–5; with ensuing discussion by Dr. Robert Bell, pp. 425–7.

present St. Clair and Detroit rivers, by which the earliest outflow of the old glacial lake probably passed southward and thence ran east as a glacial River Erie, at first tributary to Lake Lundy. As soon as that very briefly existing glacial lake was drained away, the river followed the lowest part of the shallow bed of the present Lake Erie along all its extent, which then had an eastward descent of probably 200 feet, allowing no lake or only a very small one to exist in the deepest depression of the basin; and north of Buffalo it coincided with the course of the Niagara river.

Gilbert, Wright, and Spencer, have thought that for a long time the outflow of the three great lakes above Lake Erie passed by the way of Lake Nipissing to the Mattawa and Ottawa rivers. It seems to me far more probable, however, that the epeirogenic uplift of the Nipissing region, which had elevated it already about 400 feet during the existence of Lake Warren, continued so fast that both the Trent and Nipissing-Mattawa passes were raised the additional 50 feet needed to place them above the level of Lake Algonquin before the glacial retreat uncovered the country east of them so that outlets could be obtained there.

With the continuance of the uplift of the Lake Superior basin after the formation of the Algonquin beach, the mouth of Lake Superior and the Sault Ste. Marie came into existence; and this movement allowed the lake level at Duluth to fall probably 40 or 50 feet beneath the Algonquin and present shore line. Subsequent differential elevation of the eastern and northern parts of the basin, as compared with Duluth, has again brought the west end of the lake up to the Algonquin shore, but not until the St. Louis river, while the water surface stood considerably lower than now, had deeply eroded its broad channel through the very gently sloping expanse of till from Fond du Lac to the harbor of Duluth and Superior.

The differential uplift of the Algonquin beach, as compared with Chicago and the previous mouth of Lake Warren, has been about 60 feet near the mouth of Lake Huron and at Duluth; 110 feet at the mouth of Lake Superior; 200 feet at Lake Nipissing; and 240 to 290 feet at Barrie, Lorneville, and Orillia, on Lake Simcoe. A broad lobe of the waning ice sheet, terminating on the highland area between the south end of Georgian bay and the west end of Lake Ontario, appears to have delayed the elevation of that district, so that subsequent to the formation of the Algonquin beach more uplifting took place there than at the north side of Georgian bay and about Lake Nipissing. The ascent of the Algonquin beach in nearly 200 miles from the mouth of Lake Huron northeasterly to Lake Simcoe averages about a foot per mile; and thence in

about 135 miles north to Lake Nipissing it descends at an average rate of about eight inches per mile.

While the eastern part of the Lake Algonquin area was being much uplifted, with the formation of other beaches below the first, probably the southern part of the Lake Michigan basin remained with a very slight change of attitude or none, having previously risen to approximately its present height, which it has since held with little or no change. But the northeastward elevation raising the country where Lake Algonquin and now Lake Huron have outflowed, gradually caused the water level at Chicago to rise some 40 feet above its old Algonquin level, which is shown by a sublacustrine terrace formed by the Algonquin wave erosion and beach accumulation.

On the Saugeen river, Ontario, and near the south end of Georgian bay, fresh-water shells are found in beds belonging to stages of Lake Algonquin respectively about 40 and 100 feet below the main and earliest Algonquin beach, or 90 and 78 feet above the present lake and bay.

*Lake Lundy.**—From the Forest beach at Crittenden, Erie county, N. Y., marking the latest level of Lake Warren, there is a descent of 125 feet between 860 and 735 feet above the sea to the earliest strand of the glacial Lake Lundy, which for a time occupied the northeastern three-fourths of the Lake Erie basin. A more conspicuous principal Lundy beach, 30 feet lower, on which is the "ridge road" named Lundy lane, near Niagara Falls, has an eastward ascent of 30 feet in about 40 miles from Font-hill, Ont., to Akron, N. Y., five miles north of Crittenden. Lake Lundy opened through a strait about 30 miles wide into the Lake Ontario basin. Its outflow passed eastward, across the country close north of the Finger lakes, to the Mohawk and Hudson valleys, still partly filled by the receding ice-sheet and permitting a series of mouths of Lake Lundy to be found at successively lower levels, until as the ice-border withdrew the water soon sank to the lowest point of the Ontario-Mohawk watershed at Rome, N. Y., where its level long remained, forming the Iroquois beach. One of the stages of the sinking Lake Lundy or incipient Lake Iroquois, probably nearly midway in altitude between the Lundy and Iroquois beaches, I find to be indicated by my studies of eskers in Rochester and Pittsford, N. Y.†

* J. W. Spencer, "Deformation of the Lundy Beach and Birth of Lake Erie," this Journal, III, vol. xlvii, pp. 207–211, with map, March, 1894.

† Proceedings of the Rochester Academy of Science, vol. ii, pp. 196–198, Jan., 1893.

*Lake Iroquois.**—This glacial lake, outflowing at Rome to the Mohawk and Hudson, occupied less area in the west part of the Lake Ontario basin during its earliest stage than during the later and probably longer enduring lake stage by which the high Iroquois beach in that region was formed. Previous to the date of the western development of the Iroquois beach, the early water level stood at one time only a little higher than the present Lake Ontario at Toronto and Scarboro Heights, 6 to 15 miles east of Toronto, as compared with the altitude, doubtless absolutely lower than now with regard to the sea, which the land then held in that part of the lake basin. This is shown by the occurrence of fossil fresh-water mollusks of fourteen species, and wood of ash, oak, and American yew, in beds at Toronto, described by Coleman, which now are 33 to 51 feet above Lake Ontario, or 280 to 298 feet above the sea. All the mollusk species are now living; but four are restricted, so far as known, to waters tributary to the Mississippi. A boulder-bearing surface deposit above these beds proves that the front of the ice-sheet was not far distant; but the climatic conditions of that time, clearly indicated by the fauna and flora, were as mild as now. There next ensued, probably, a gradual rise of the lake, due to an uplifting of the country about its outlet at Rome, until it stood at the level of the well defined Iroquois beach, which has a height at Toronto of about 200 feet above Lake Ontario. Thick fossiliferous delta deposits had been, meanwhile, brought into the north edge of the lake at Toronto and several miles eastward along the lake-cliff section of Scarboro Heights, described by Hinde; and repeated re-advances of the ice-front, one during, and another after, the delta accumulation, formed, at the locality last noted, two deposits of till or boulder-clay.

In a limited sense the Toronto and Scarboro fossils may be called Interglacial, since they lie between deposits of glacial drift; but they seem better referred to moderate oscillations of the ice boundary than to the distinct glacial epochs which Coleman and Hinde infer from them. Both these beds and

* J. W. Spencer, "The Deformation of the Iroquois Beach and Birth of Lake Ontario," this Journal, III, vol. xl, pp. 443–451, with map, Dec., 1890; and papers previously cited. Thomas Roy (in paper by Sir Charles Lyell), Proceedings Geol. Soc., London, vol. ii, 1837, pp. 537, 538. Sir Charles Lyell, Travels in N. A., in 1841–42, vol. ii, chapter xx. E. J. Chapman, Canadian Journal, new series, vol. vi, 1861, pp. 221–229, and 497, 498. Sandford Fleming, Can. Jour., same vol. vi, pp. 247–253. George J. Hinde, Can. Jour., vol. xv, 1877, pp. 388–413. A. P. Coleman, Am. Geologist, vol. xiii, pp. 85–95, Feb. 1894. Geol. Survey of Canada, Report of Progress to 1863, pp. 912, 913. James Hall, Geology of New York, Part iv, 1843, pp. 348–351. Baron Gerard de Geer, "On Pleistocene Changes of Level in eastern North America." Proc. Boston Soc. Nat. Hist., vol. xxv, 1892, pp. 454–477, with map; also (excepting the map) in Am. Geologist, vol. xi, pp. 22–44, Jan., 1893. G. K. Gilbert, F. B. Taylor, E. W. Claypole, G. F. Wright, and Warren Upham, as cited for Lakes Warren and Algonquin.

the richly fossiliferous Leda clays, which last overlie the latest glacial drift in the St. Lawrence, Ottawa, and Champlain valleys, may be referred to the closing stage or Champlain epoch of the Ice age; and they both testify, like the partially forest-covered Malaspina ice-sheet in Alaska, of the close sequence of a warm climate, with luxuriant plant and animal life, during and immediately after the recession of the ice-sheet. The transition from the Glacial to the Champlain climate seems readily explained by the epeirogenic depression which ended the Glacial period.*

The height of Lake Ontario is 247 feet; and that of the old Iroquois outlet crossing the water-shed at Rome is 440 feet, above the sea level. Thence the Iroquois beach in its course northward adjacent to the eastern end of Lake Ontario has a gradual ascent of about five feet per mile along a distance of 55 miles to the latitude of Watertown, where the highest beach is 730 feet above the sea, showing that a differential uplift of about 290 feet has taken place, in comparison with the Rome outlet. From Rome westward to Rochester, the beach has nearly the same height with the outlet; but farther westward it descends to 385 feet above the sea at Lewiston and 363 feet at Hamilton, at the western end of Lake Ontario. Continuing along the beach north of the lake, the same elevation as the Rome outlet is reached near Toronto, and thence east-northeastward an uplift is found, similar to that before described east of the lake, its amount near Trenton and Belleville above Rome being about 240 feet. It is to be added that northward from Rome the Iroquois beach becomes divided into a series of distinct beaches, marking stages in the northeastward rise of the land and having near Watertown a vertical range of 80 feet below the highest and oldest, which was before noted; and that westward a similar series of strand lines also lies below the highest, likewise before noted, which there, however, contrary to the order northeastward, was the newest. The highest beach near Watertown was probably contemporaneous with the fossiliferous beds of Toronto; some of the intermediate northeastern beaches corresponded to the delta deposits of Scarboro; and the lowest northeastward lake level was continuous with the highest at Toronto, Hamilton, Lewiston, and east to Rome.

Between Lakes Warren and Lundy the old water level near the west end of Lake Ontario fell 125 feet, minus some amount to be subtracted for the progressing northeastward elevation of the land. The two Lundy shores are 30 feet apart verti-

* J. D. Dana, Trans. Conn. Acad. of Arts and Sciences, vol. ii, 1870, p. 67; this Journal, III, vol. x, pp. 168–183, Sept., 1875. Warren Upham, Glacialists' Magazine, vol. i, pp. 236–240, June, 1894.

cally. From the lower and main Lundy beach the water fell about 480 feet to the earliest stage of Lake Iroquois when the Toronto fossil shells lived in the edge of that lake, excepting that here again some undetermined amount must be subtracted to compensate the concurrent rise of the land. Adding these vertical intervals together, we have 635 feet, which probably may be reduced 100 feet, more or less, for the effects of the accompanying epeirogenic uplift. We have left some 500 or 550 feet, to be subtracted from the altitude of the old Chicago outlet of Lake Warren, believed to have been then approximately as now, 590 feet above the sea, to give the earliest altitude of the Rome outlet. It thus appears, as I concluded from a similar computation four years ago, that the Rome outlet was at first only 50 or 100 feet above the sea level.* It was gradually uplifted, participating in the differential rise of the whole Ontario basin, to about 300 feet above the sea while the outflow continued here, and to probably 350 feet or more, lacking less than 100 feet of its present height, by the time when the much farther retreat of the ice permitted the extension of the sea to Ogdensburgh and Brockville, on the St. Lawrence river near the mouth of Lake Ontario. Intermediate between Lake Iroquois and the Champlain incursion of the sea, the glacial Lake St. Lawrence, into which Lake Iroquois was merged by the retreat of the ice-sheet from the northern side of the Adirondacks, filled the Lake Ontario basin for a considerable time at levels below the Iroquois beaches.

As the area of Lake Warren was being differentially much elevated during the earlier existence of that lake, and as the area of Lake Algonquin was similarly uplifted in part or wholly contemporaneously with the Iroquois basin, so this region was being rapidly raised and tilted upward to the north and east while the lake level, held constantly without important downward cutting at the Rome outlet, inscribed many shore lines on the slowly moving land. All the movement throughout the whole region probably was upward; but the position of Rome, and its greater rise than western parts of the basin during the existence of Lake Iroquois, caused the old beaches westward to have now declining gradients.

Lake Hudson-Champlain.†—The absence of marine fossils

* Bulletin Geol. Soc. Am., vol, ii, pp. 260–262.

† Warren Upham, Bulletin Geol. Soc. Am., vol. i, p. 566; vol. ii, p. 265; vol. iii, pp. 484–487 (first using this name). C. H. Hitchcock, Geology of Vermont, 1861, vol. i, pp. 93–167, with map. J. S. Newberry, Pop. Sci. Monthly, vol. xiii, 1879, pp. 641–660. F. J. H. Merrill, this Journal, III, vol. xli, pp. 460–466, June, 1891. W. M. Davis, Proc. Boston Soc. Nat. Hist., vol. xxv, 1891, pp. 318–334. S. Prentiss Baldwin, "Pleistocene History of the Champlain Valley," Am. Geologist, vol. xiii, pp. 170–184, with map, March, 1894. Baron de Geer, as cited for Lake Iroquois.

in beds overlying the glacial drift on the shores of southern New England, Long Island, and New Jersey, and the water-courses which extend from the terminal moraine on Long Island southward across the adjacent modified drift plain and continue beneath the sea level of the Great South bay and other bays between the shore and its bordering long beaches, prove that this coast stood higher than now when the ice-sheet extended to its farthest limit. A measure of this elevation of the seaboard in the vicinity of New York during the Champlain epoch is supplied, as I believe, by the shallow submarine channel of the Hudson, which has been traced by the soundings of the U. S. Coast Survey from about 12 miles off Sandy Hook to a distance of about 90 miles southeastward. This submerged channel, lying between the present mouth of the Hudson and the very deep submarine fjord of this river, ranges from 10 to 15 fathoms in depth, with an average width of $1\frac{1}{4}$ miles, along its extent of 80 miles, the depth being measured from the top of its banks, which, with the adjacent sea-bed, are covered by 15 to 40 fathoms of water, increasing southeastward with the slope of this margin of the continental plateau. During the whole or a considerable part of the time of the glacial Lake Iroquois, this area stretching 100 miles southeastward from New York was probably a land surface, across which the Hudson flowed with a slight descent to the sea. But northward from the present mouth of the Hudson the land at that time stood lower than now; and the amount of its depression, beginning near the city of New York and increasing from south to north, as shown by terraces and deltas of the glacial Lake Hudson-Champlain, which were formed before this long and narrow lake became merged in the glacial Lake St. Lawrence, was nearly 180 feet at West Point, 275 feet at Catskill, and 340 feet at Albany and Schenectady. From these figures, however, we must subtract the amount of descent of the Hudson river, which in its channel outside the present harbor of New York may probably have been once 50 or 60 feet in its length of about 100 miles.

Before the time of disappearance of the ice-barrier from the St. Lawrence valley at Quebec, the descent of the Hudson river beyond New York city may have diminished, or the seaboard at New York may have sunk so as to bring the shore line nearly to its present position; but the Hudson valley meanwhile had been uplifted, so that the outflow from the Lake St. Lawrence crossed the low divide, now about 150 feet above the sea, between Lake Champlain and the Hudson. This is known by the extension of fossiliferous marine deposits along the Lake Champlain basin nearly to its southern end, while they are wholly wanting along all the Hudson valley.

Indeed, the outflowing river from Lakes Iroquois, Hudson-Champlain, and St. Lawrence, or the Hudson during the Post-glacial period, channeled the lower part of this valley to a depth of about 100 feet below the present sea level, proving that the land there, as Merrill points out, stood so much higher than now at some time after the ice retreated.

According to the observations of Davis, Baldwin, and Baron de Geer, the highest shore line of the Lake Hudson-Champlain is now elevated to about 275 feet above the sea at Catskill, N. Y.; 550 feet in Chesterfield, N. Y., on the west side of Lake Champlain opposite to Burlington; and 658 feet at St. Albans, Vt. Assuming that the mouth of the lake, near New York city, was 50 feet above the sea, the differential northward uplift of the originally level shore has been at the rate of about two feet per mile for the 100 miles from the present mouth of the Hudson to Catskill; 1·7 feet per mile for the next 160 miles north to Chesterfield; and about three and a half feet per mile in the next 30 miles north-northeastward to St. Albans. Perhaps a higher beach may exist in Chesterfield, which would bring these gradients nearer to uniformity. The series noted there by Baldwin comprises eight beaches referable to the successive water levels of Lake Hudson-Champlain, Lake St. Lawrence, and the sea in the Champlain basin, their heights above the sea level of to-day being 550 feet, 530, 470, 423, 386, 365, 335, and 290 feet. The mean level of Lake Champlain is 97 feet above the sea, and its maximum depth 402 feet. The lower four of these beaches belonged to the Champlain arm of the enlarged Gulf of St. Lawrence, as shown by the height of its sand deltas and associated fossiliferous clays; but the higher four represent stages of the Lakes Hudson-Champlain and St. Lawrence. These shore lines, like those of the glacial lakes farther west to Lake Agassiz, were probably formed during times of rest or slackening in the somewhat intermittent epeirogenic elevation of the land.

*Lake St. Lawrence.**—The records of the Glacial and Champlain epochs in the St. Lawrence valley have been most fully studied during many years by Sir William Dawson, to whose work chiefly we are indebted for detailed descriptions of the evidences of the marine submergence of that region to a maxi-

* Sir J. William Dawson, The Canadian Ice Age (Montreal, 1893), p. 301, with maps and sections, views of scenery, and nine plates of Pleistocene fossils. This volume sums up the author's work since 1855 on the glacial drift and associated lacustrine and Champlain marine formations of the St. Lawrence valley, embodying the studies which had been published in many papers in the "Canadian Naturalist and Geologist" and elsewhere. He had given a similar summary in a pamphlet of 112 pages, "Notes on the Post-pliocene of Canada," in 1872. J. W. Spencer, G. K. Gilbert, Baron de Geer, S. Prentiss Baldwin, and Warren Upham, as before cited for Lakes Warren, Algonquin, Iroquois, and Hudson-Champlain.

mum height at Montreal somewhat exceeding 500 feet above the present sea level. Earlier than that time of occupation of the depressed broad valley by the sea, it was filled from Lake Ontario to near Quebec, by a great glacial lake, held on its northeast side by the receding continental ice-sheet. The directions of the glacial striæ and transportation of the drift in the St. Lawrence valley, running southwestward at Montreal and onward to the great lakes, but eastward from Quebec down the shores of the Gulf of St. Lawrence, and southeast across Nova Scotia and New Brunswick, show that the latest remnant of the ice barrier blockading this valley was melted away in the neighborhood of Quebec, then admitting the sea to a large, low region westward. Until this barrier was removed, a glacial lake, which here for convenience of description and citation is designated as the Lake St. Lawrence, dating from the confluence of Lakes Iroquois and Hudson-Champlain and growing northward and eastward, spread over the Ottawa valley probably to the mouth of the Mattawa, and down the St. Lawrence, as fast as the ice-front was melted back.

When Lake Iroquois ceased to outflow at Rome and, after intervening stages of outlets existing for a short time at successively lower levels north of the Adirondacks, began to occupy the Champlain basin and the St. Lawrence valley northward, changing thus to the Lake St. Lawrence, its surface fell by these stages about 250 feet to the glacial Lake Hudson-Champlain, which had doubtless reached northward nearly to the St. Lawrence. After this reduction of the water body in the Ontario basin, it still had a depth of about 150 feet over the present mouth of Lake Ontario, as shown by a beach traced by Gilbert, which thence rises northeastward but declines toward the south and southwest. Its plane, which is nearly parallel with the higher Iroquois beaches, sinks to the present lake level near Oswego, N. Y. Farther southwestward the shore of the glacial lake at this lower stage has been since submerged by Lake Ontario. The Niagara river was then longer than now, and the lower part of its extent has become covered by the present lake. From the time of the union of Lakes Iroquois and Hudson-Champlain, a strait, at first about 150 feet deep, but later probably diminished on account of the rise of the land to a depth of about 50 feet, joined the broad expanse of water in the Ontario basin with the larger expanse in the St. Lawrence and Ottawa valleys and the basin of Lake Champlain. At the subsequent time of ingress of the sea past Quebec the level of Lake St. Lawrence fell probably 50 feet or less to the ocean level. The place of the glacial lake so far westward as the Thousand Islands was then taken by the sea,

with the marine fauna which is preserved in the Leda clays and Saxicava sands.

The Champlain Marine Submergence.

That the land northward from Boston was lower than now while the ice-sheet was being melted away, is proved by the occurrence of fossil mollusks of far northern range, including *Leda arctica* Gray, which is now found living only in the Arctic seas, preferring localities which receive muddy streams from existing glaciers and from the Greenland ice-sheet. This species is plentiful in the stratified clays resting on the till in the St. Lawrence valley and in New Brunswick and Maine, extending southward to Portsmouth, N. H. But it is known that the land was elevated from this depression to about its present height before the sea here became warm and the southern mollusks, which exist as colonies in the Gulf of St. Lawrence, migrated thither, for these southern species are not included in the extensive lists of the fossil fauna found in the beds overlying the till.

In the St. Lawrence basin these marine deposits reach to the southern end of Lake Champlain, to Ogdensburgh and Brockville, and at least to Pembroke and Allumette island, in the Ottawa river, about 75 miles above the city of Ottawa. The isthmus of Chiegnecto, connecting Nova Scotia with New Brunswick, was submerged, and the sea extended 50 to 100 miles up the valleys of the chief rivers of Maine and New Brunswick. The uplift of this region from the Champlain sea level was 10 to 25 feet in the vicinity of Boston and northeastward to Cape Ann; about 150 feet near Portsmouth, N. H.; from 150 to about 300 feet along the coast of Maine and southern New Brunswick; about 40 feet on the northwestern shore of Nova Scotia; thence increasing westward to 200 feet in the Bay of Chaleurs, 375 feet in the St. Lawrence valley opposite the Saguenay, and about 560 feet at Montreal; 150 to 400 or 500 feet, increasing from south to north, along the basin of Lake Champlain; about 275 feet at Ogdensburgh, and 450 feet near the city of Ottawa. The differential elevation was practically completed, as we have seen from the boreal character of the Champlain marine molluscan fauna, shortly after the departure of the ice-sheet. With the areas of the glacial Lakes Agassiz, Warren, and Iroquois, in the interior of the continent, this coastal region gives testimony of a wave-like epeirogenic elevation of the formerly ice-laden portion of the earth's crust, proportionate with the glacial melting and closely following the retreat of the ice from its boundaries of greatest

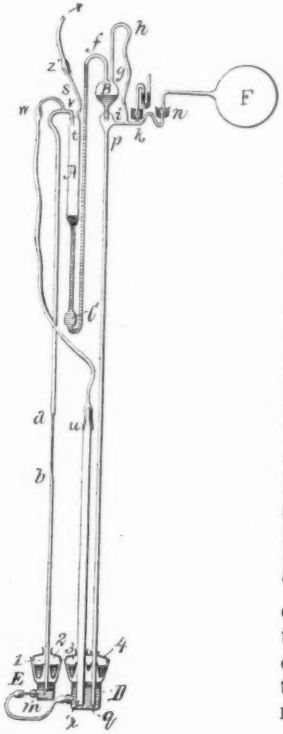
extent inward to the areas on which its waning remnants lingered the latest.

On the Green Mountains of Vermont, the White Mountains region, and indeed probably over a large part of New England, a tract of the departing ice-sheet remained after the access of the sea to the St. Lawrence basin left the New England ice as an isolated mass. This is known by the large tribute of stratified drift quickly brought by streams from the melting ice of the Green Mountains area and deposited as gravel and sand deltas and offshore clays of the Winooski, LaMoille, and Missisquoi rivers, described by Hitchcock and Baldwin, in the east border of the Champlain arm of the sea. On the west, too, a considerable remnant of the ice-sheet seems to have remained unmelted until this time on the Adirondacks, and to have likewise supplied the deltas and marine clays of the Au Sable, Saranac, and Chazy rivers in New York. Deflections of glacial striation down the valleys, with corresponding drift transportation and formation of local moraines across some of the mountain valleys, have been recorded by Hitchcock, Stone, and others, in Vermont and New Hampshire; but the time allowed for such glacial action, under the warm Champlain climate, was very short. The earlier melting of the ice along the St. Lawrence valley than on these mountain tracts was due on one side to the laving action of the waves of Lakes Iroquois and St. Lawrence, and on the other side to the washing of the ice-cliffs by the fast encroaching sea in the Gulf of St. Lawrence, until at last near Quebec the barrier was severed.

From the Champlain submergence our Atlantic coast was raised somewhat higher than now; and its latest movement from New Jersey to southern Greenland has been a moderate depression. The vertical amount of this postglacial elevation above the present height, and of the recent subsidence, on all the coast of New Jersey, New England, and the eastern provinces of Canada, is known to have ranged from 10 feet to a maximum of at least 80 feet at the head of the Bay of Fundy, as is attested in many places by stumps of forests, rooted where they grew, and by peat beds now submerged by the sea. As in Scandinavia, the restoration of isostatic equilibrium is attended by minor oscillations, the conditions requisite for repose having been overpassed by the early reëlevation of outer portions of each of these great glaciated areas. The close of the Ice age was not long ago, geologically speaking, for equilibrium of the disturbed areas has not yet been restored.

ART. II.—*An Automatic Mercury Vacuum Pump;* by M. I. PUPIN, PH.D., Columbia College.

THE pump which forms the subject of the following description is a combination of two distinct forms of apparatus. First, a *suction pump* capable of raising mercury to practically any height and secondly, an *ordinary Sprengel pump.* The part connecting the two is a syphon barometer, properly disposed with respect to the two parts which it connects.



Referring now to the diagram I shall describe each part separately and shall then explain the modus operandi of the combination.

1. *The Suction Pump.*

It consists of the reservoir A to which are joined the short tube *n* and the tube *t v m* which I shall call *the suction tube.* A short branch tube *v w* is connected by a rubber tube *w u* to the wide tube *u x*. This wide tube I shall call the *valve tube.* The suction tube and the valve tube dip in two separate mercury vessels E and D which are provided with specially constructed glass dishes 1, 2, 3, 4 containing concentrated sulphuric acid for drying purposes. The two vessels are connected to each other by means of a rubber tube. A part of the suction tube *a b* about 20^{cm} long has a cross-section one-half as large as that of the rest of the tube.

2. *The Sprengel Pump.*

It is of the ordinary type and consists of the reservoir B from which the mercury drops into a long tube *p q, the tube of descent.* This tube may be given any convenient length; 160^{cm} will be found sufficient for rapid working. The tube of descent carries a lateral extension which is connected to the reservoir B by means of the tube *g h i.* The object of this

connection is to keep the gas pressure above and below the mercury in the reservoir the same. Two ground joints *k* and *n* are made air-tight by the mercury which fills the surrounding hoods. These joints connect this extension to a manometer and the vessel F which is to be exhausted.

3. *The Syphon barometer connection.*

This is the part consisting of the bulb *c* and the tube *c f*. The length of this tube is about 80^{cm}.

All the parts are made of glass.

The modus operandi.

First a little mercury is poured through *z s* into A until the reservoir *c* is about half full, which is considerably more than sufficient to fill the tube *c f*. By means of a rubber tube, *z s* is connected then to a water-pump or any other suction pump that may be available. Suppose that this auxiliary suction pump is capable of reducing the pressure in A to say 40^{mm} and suppose also that the barometric pressure is 760^{mm}. Owing to the action of the auxiliary suction pump the gas pressure in A is continually reduced and therefore also in F and in all other parts connected with A. Mercury rises in the valve and suction tubes and also in the tube of descent. The extremity *m* of the suction tube is placed at such a distance below the initial level of the mercury in E and D that when the mercury column in the suction tube is about 60^{cm} long the mercury level in E (which sinks rapidly on account of the rising of the mercury into the valve tube) has just reached *m*. From that moment on no more mercury gets into the suction tube. But owing to the action of the auxiliary pump the pressure in A is being still reduced, hence the column in the valve tube rises still higher and the level in E sinks still lower. In the mean time the column in the suction tube rises bodily owing to the external air pressure until it reaches the narrow part *a b* when it begins to lengthen out, and since *a b* is 20^{cm} long and the initial length of the mercury column is 60^{cm} it follows that this column will be lengthened out to 70^{cm} and no more. Hence as soon as the pressure in A has been reduced by 70^{cm} this column will rise with accelerated velocity until it is injected by the external air pressure into the reservoir A. The external air rushes then into A and through *t v w* into the valve tube. The valve tube column sinks and the level in E rises. But it will rise more rapidly in D than in E owing to the friction of the narrow rubber tube connecting the two vessels E and D. Hence it will continue to rise for a short time even after it has reached *m* and by closing the suction tube started again the action of the auxiliary suction pump.

This retardation of the level in E is of considerable importance, for if the suction tube had no contraction *a b* and if the two levels in E and D were continually of the same height then the mercury would be sucked up through the suction tube not in form of solid columns but in form of numerous drops separated from each other by air bubbles. This would render the rapidity of action less satisfactory; besides, it would also cause a rapid oxidation of the mercury.

As soon as the quantity of mercury injected into A brings the distance between the level in A and the point *f* into the vicinity of the barometric height then the mercury begins to overflow from the syphon tube into the reservoir B and the exhausting of F begins. By squeezing the tube connecting E and D the rapidity of supply to B is varied, hence the quantity of mercury in B can thus very easily be kept within certain desirable limits.

The simplicity and the convenience of the apparatus need no comment. Suffice it to observe that it has no stopcocks and that it can operate with a much smaller quantity of mercury than required by ordinary mercury pumps.

My experiments with vacuum tube discharges suggested long ago to my mind a pump of this type; but want of time and of a glass blower at a convenient distance prevented me from giving my ideas on this matter a practical test, until last summer.* I intend to publish soon numerical data concerning the rapidity of working of the various forms of pumps of this type. The vacua obtainable by it are, of course, the same as those obtainable by the ordinary form of the Sprengel pump.

Electrical Laboratory, Dec. 17th, 1894. Columbia College, New York.

Art. III.—*On Graphical Thermodynamics;* by René de Saussure.

Translated by the author from vol. xxxi of the *Archives des Sciences physiques et naturelles*, May, 1894.

1. Heat is usually regarded as a periodical motion of the particles constituting the material bodies; if this be true, the variations in the physical state of a substance are due to the variations of the state of this periodical motion; in other words, the physical state of the substance is a function of the state of the periodical motion. Since the periodical motion of the particles can be defined by its kinetic energy and by the

*[More particular details of construction will be given to Messrs. Eimer & Amend, 18th Street and 3d Ave., New York, and to Herr Kramer, Glasbläser, Fridrich Str., Freiburg, Baden]

duration of its period, the physical state of the substance can be completely defined by means of these two data, provided that the weight of the substance remains the same.

It is by so defining the state of a body, that Clausius succeeded in demonstrating the fundamental theorems of thermodynamics, with the help only of the laws of mechanics, without making any hypothesis as to the form of the trajectories described by the particles of the body.

But if it is desired to establish the theory of the transformations which take place in a substance under the influence of heat, it is necessary to define the nature of the periodical motions, as well as in the theory of light. We can assume for instance, that this motion is a straight vibratory motion on either side of a fixed center.

By this hypothesis, we still need two data to define completely the motion of the particles, i. e., the amplitude a and the duration i of one period of the vibratory motion. And since the state of the substance (whose mass is taken as the unit) is a result of the state of motion of the particles, the state of said substance can be considered as a function of the two variables a and i.

2. On the other hand, to define the physical state of a substance by means of experimental data, the variables used are: the volume V, the absolute temperature T and the outside pressure P (the mass being still taken as the unit).*

Hence, the state of a body can also be regarded as a function of the three variables: P, V and T. But these variables are not independent, that is to say: the same weight of the same body cannot occupy the same volume at the same pressure and at different temperatures, since two variables are sufficient to define the state of the substance. For each body, there is a relation $F(P, V, T) = 0$ known as the *equation* of said body, so that the value of either of the three variables P, V or T is a direct result of the values attributed to the two others.

Considering P, V and T as three coördinates, the equation $F(P, V, T) = 0$ represents a surface, any point of which corresponds to a certain state of the body. This surface is therefore a "representative locus" of the different states under which the body can exist, and is known as the "*thermodynamic surface.*"

3. When the physical state of a substance is defined by the first method, i. e., by means of the amplitude a and of the period i of the vibratory motion which constitutes the heat, the variables a and i can also be treated as two coördinates,

* In the following study, we assume that all the particles of the body are at the same temperature, i. e., that they all have an identical vibratory motion.

and the physical state can be represented on a piece of paper by the point corresponding to these coördinates. In this case, the piece of paper itself or a part of it, is the representative locus of the different states under which the substance can exist.

It follows that each point of the thermodynamic surface $F(P, V, T) = 0$ corresponds to a point on the sheet of paper, and conversely. If the variables P, V and T vary continuously, the variables a and i shall also vary continuously, since the variation of the state of the body is itself continuous; so that the coördinates P, V, T are continuous functions of the coördinates a and i.

$$\begin{cases} P = \lambda(a, i) \\ V = \mu(a, i) \\ T = \upsilon(a, i) \end{cases} \qquad (1)$$

These three equations can be regarded as the general equation to the thermodynamic surface in terms of two auxiliary variables a and i; hence, by eliminating a and i between them, the result must be: $F(P, V, T) = 0$.

The variables a and i can be considered as the coördinates of any point on the thermodynamic surface. Any relation between a and i represents a curve traced on this surface, i. e., a cycle of transformations undergone by the substance. We have just seen that the functions λ, μ, υ must be such as to lead to the relation $F(P, V, T) = 0$ by eliminating a and i between equations (1); but as long as these functions are submitted only to this condition, the variables a and i are still arbitrary variables and do not necessarily denote the amplitude and the period of the vibratory motion, since there are an infinite number of ways of representing the same surface by means of two auxiliary variables. For instance, the equation of the body: $T = f(P, V)$ can be put under the form:

$$\begin{cases} P = u \\ V = v \\ T = f(u, v) \end{cases}$$

u and v being the two auxiliary coördinates chosen to represent graphically the cycles of transformations.

These coördinates u and v being equal to P and V respectively, the graphical representation thus obtained would be the same as the one first introduced in thermodynamics by Clapeyron, and would have the same property, i. e., the area lying between the axis of V, two ordinates and the path described by the body would be equal to the external work.

If other coördinates are chosen, the properties of the graphical representation will change, for it is evident that these

properties depend essentially upon the choice of the two auxiliary variables a and i; therefore, they ought to be chosen in such a way as to give the best possible graphical representation of the cycles of transformation, i. e., in such a way as to enable us to determine graphically the greatest possible number of the physical elements depending upon the transformation, by means of geometrical magnitudes depending only upon the form and position of the path described by the body in the adopted system of coördinates. Before defining this system, let us examine what conditions must be fulfilled by the functions λ, μ, υ, in order that the auxiliary variables a and i be respectively the amplitude and the period of the vibratory motion.

4. Denoting by m the mass of one of the particles composing the substance and by u the mean velocity of the vibratory motion, the expression $\frac{1}{2}\Sigma mu^2$ is the actual kinetic energy of the heat (the sum Σ being extended to all the particles). Dividing this sum by the mechanical equivalent of heat E, the result is equal to the amount of heat actually contained in the body.

This amount of heat is proportional to the absolute temperature, hence:

$$\tfrac{1}{2}\Sigma mu^2 = \mathrm{KTE} \qquad (2)$$

K being a constant.

Denoting by f the mean value of the force producing the vibratory motion, the formulæ:

$$fa = mu^2$$
$$f = 2\pi^2 m \frac{a}{i^2} \qquad (3)$$

can be established without difficulty, since in all vibratory motions of small amplitude, the force producing the vibration is proportional to the displacement of the particles.

Combining equations (2) and (3) and noticing that $\Sigma m = 1$ and that the mean velocity u is the same for all the particles of the substance, we shall obtain:

$$\mathrm{T} = \frac{\pi^2}{\mathrm{KE}} \frac{a^2}{i^2} \qquad (4)$$

which is the expression of T in terms of a and i, and is therefore identical to the third of equations (1); in other words, when the two auxiliary variables a and i denote the amplitude and the period of the vibratory motion, the function v is no longer arbitrary, and the equation to the thermodynamic surface is:

$$\begin{cases} \mathrm{P} = \lambda(a, i) \\ \mathrm{V} = \mu(a, i) \\ \mathrm{T} = \dfrac{\pi^2}{\mathrm{KE}} \dfrac{a^2}{i^2} \end{cases} \qquad (5)$$

the functions λ and μ being still submitted to the condition that the result of eliminating a and i between equations (5) be: $F(P, V, T) = 0$.

When the functions λ and μ have been determined for a particular substance, equations (5) do not only represent the thermodynamic surface, but also the value of the two elements (a and i) of the vibratory motion, corresponding to any state of the substance defined by experimental data (P, V, T).

The last of equations (5) is the same for all substances, except that the value of the constant K changes from one substance to another. The determination of the functions λ and μ will be investigated after we shall have studied the properties of the graphical representation, which properties can be found by assuming that these functions are known.

5. When a substance undergoes an elementary and reversible transformation,* the amount of heat dH, absorbed by the unit of mass, is composed of two parts: the variation of the actual energy of the heat contained in the substance, and the amount of heat absorbed by the total work (external and internal).

The first part is the elementary variation of the expression $\frac{1}{E}\Sigma\frac{1}{2}mu^2$, as found above; the second is the heat absorbed by the work done by the force f for a variation da of the amplitude. Hence:

$$EdH = d\Sigma\tfrac{1}{2}mu^2 + \Sigma f da$$

But, by differentiating equation (2):

$$d\Sigma\tfrac{1}{2}mu^2 = KEdT$$

This relation shows that the constant K is the quotient of the variation of the actual amount of heat contained in the substance, by the corresponding variation of temperature, so that K is by definition the *absolute specific heat* of the substance.

We have also, from preceding formulæ:

$$\Sigma f da = \Sigma mu^2 \frac{da}{a} = 2KTE\frac{da}{a}$$

Whence finally

$$dH = KdT + 2KT\frac{da}{a}$$

Such is the expression of dH in terms of T and a; equation (4) gives by differentiation:

$$\frac{da}{a} = \frac{di}{i} + \tfrac{1}{2}\frac{dT}{T}$$

* The formulæ contained in this paragraph have been already established in "La Thermodynamique et ses principales applications," by J. Moutier, Paris, 1885; we recall them here, as we will have to use them in some of the demonstrations.

By the aid of this equation and of the preceding one, the value of dH can also be obtained in terms of T and i or of a and i:

$$\begin{cases} \frac{d\mathrm{H}}{\mathrm{T}} = \mathrm{K}\left(\frac{d\mathrm{T}}{\mathrm{T}} + 2\frac{da}{a}\right) \\ \frac{d\mathrm{H}}{\mathrm{T}} = 2\mathrm{K}\left(\frac{d\mathrm{T}}{\mathrm{T}} + \frac{di}{i}\right) \\ \frac{d\mathrm{H}}{\mathrm{T}} = 2\mathrm{K}\left(2\frac{da}{a} - \frac{di}{i}\right) \end{cases} \tag{6}$$

6. The two variables, which we intend to take as coördinates in this graphical study, depend directly upon the amplitude a and the period i of the vibratory motion of heat; denoting these variables by φ and s, we shall define them by the equations:

$$\begin{cases} \varphi = \frac{\pi}{i^2} \\ \mathrm{S} = \pi a^2 \end{cases} \tag{7}$$

By the aid of equation (7), any of the formulæ given above and involving a and i, can be transformed into corresponding formulæ involving φ and s.

For instance, by solving equations (7) with respect to a and i, and substituting the result in equations (5), we shall obtain the equation to the thermodynamic surface in terms of φ and s, as follows:

$$\begin{cases} \mathrm{P} = f(\varphi, s) \\ \mathrm{V} = g(\varphi, s) \\ \mathrm{T} = \frac{1}{\mathrm{KE}}\varphi s \end{cases} \tag{8}$$

All the other equations can be transformed in the same manner, so that it is understood that the two independent variables are now φ and s, and that these two quantities shall be taken as the coördinates of the point representing the physical state of the substance.

As these variables have been defined in an arbitrary manner, let us first investigate their physical nature. To reach this end, we must consider the vibratory motion of the particles as the projection of a uniform circular motion on one of its diameters; the radius of the circle is then equal to the amplitude of the vibration, and the velocity of the uniform motion is equal to the maximum velocity of the vibratory motion. It can readily be seen, that the centripetal force of the circular motion is equal to the mean value f of the force supposed to produce the vibratory motion.

The total work (external and internal) absorbed by the substance during an elementary transformation is $\Sigma f da$ as seen above. We can write identically:

$$fda = \left(\frac{f}{2\pi a}\right)(2\pi a da)$$

Let $\frac{f}{2\pi a} = \varphi_m$. Since $2\pi a$ is equal to the length of the circumference, φ_m represents geometrically the value of the centripetal force referred to the unit of length, i. e., a pressure, of so many pounds per foot, supposed to be acting on the circumference of the circle.

Let $2\pi a da = ds$ or $\pi a^2 = s$. Then s is the area of the circle.

According to these definitions:

$$\Sigma f da = \Sigma \varphi_m ds$$

If now in the equation $\varphi_m = \frac{f}{2\pi a}$ f be replaced by its value as given in equation (3), the result is:

$$\varphi_m = \frac{\pi m}{i^2}$$

and since $\Sigma m = 1$:

$$\Sigma \varphi_m = \frac{\pi}{i^2} = \varphi$$

Whence:

$$\Sigma f da = \Sigma(\varphi_m ds) = (\Sigma \varphi_m) ds = \varphi ds$$

With:

$$\varphi = \frac{\pi}{i^2} \text{ and } s = \pi a^2$$

The last two equations are precisely the ones by which φ and s have been first defined.

Since $\varphi = \varphi_m$ when $m = 1$, and since φ_m is the pressure supposed to be acting on the circumference of the circle corresponding to the particle of mass m, we can define the physical nature of φ and s as follows: If the unit of mass of a substance be represented geometrically or symbolically by a circle, the physical state of said substance can be completely defined by the area s of the circle and by a pressure φ, supposed to be acting on the circumference of the circle. The two data, thus defining the state of the substance, are precisely the coördinates φ and s, which determine the position of the point representing this state. For this reason, the abscissa s shall be called the "*symbolical volume*" and the ordinate φ the "*symbolical pressure*" of the substance.

As the considerations developed in this paragraph are somewhat abstract, it must not be forgotten that the graphical method, which is the object of this study is quite independent of these theoretical considerations, since the two variables φ

and s can always be regarded as two variables defined by the equations: $\varphi = \frac{\pi}{i^2}$ and $s = \pi a^2$, whatever be their physical nature; moreover, we shall find other reasons for regarding φ as a pressure and s as a volume.

Properties of the graphical method.

7. Let M be the point representing any physical state of a substance, and φ and s its coördinates, then according to the previous definitions:

$$a = \sqrt{\frac{s}{\pi}} \text{ and } i = \sqrt{\frac{\pi}{\varphi}}$$

So that a and i, hence the state of the vibratory motion, are readily obtained from the actual value of the coördinates of point M.

Denoting by R the total work absorbed during a transformation, we have found that:

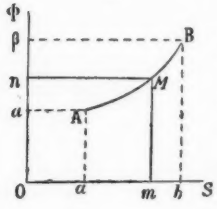
$$dR = \Sigma f da = \varphi ds$$

Whence by integration:

$$R = \int_a^b \varphi ds$$

i. e., if AB be the curve (fig. 1) representing the path of the substance referred to the coördinates φ and s, the total work (external and internal) absorbed during the transformation is equal to the area $AabB$ limited by the path, the axis of s and the two extreme ordinates.

1.



Comparing this result with the property of Clapeyron's graphical method, we see that the symbolical pressure and the symbolical volume are in the same relation with the total work, as the ordinary pressure and volume are with the external work.

8. The last of equations (8):

$$\varphi s = \text{KTE}$$

holding true for all substances, shows that the area of the rectangle $MmOn$ formed by the coördinates φ and s, is equal to the actual amount of energy contained in the substance at the physical state M. Thus, if AB represents the path of the substance the area of the rectangles $AaOa$ and $BbO\beta$ is equal to the energy of the heat contained in the substance at its initial and final states.

Since the energy of the heat: KTE is proportional to the temperature T, we can also say that the area of the rectangle MO is proportional to the temperature of the substance at the state M.

When a substance undergoes a transformation, its temperature being maintained constant, the second member of the equation $\varphi s = \text{KTE}$ remains constant. Hence, the general equation of the isothermal lines is:

$$\varphi = \text{constant}$$

which is the equation of equilateral hyperbolas, whose asymptotes coincide with the axes of coördinates. The isothermal lines are the same for all substances, since the equation: $\varphi s = \text{KTE}$ applies to any substance.

Remark: As $\varphi s = \pi^2 \frac{a^2}{i^2} = \text{KTE}$, we see that the ratio of the square of the amplitude to the square of the period of the vibratory motion of the heat, is proportional to the temperature; so that this ratio remains constant as long as the temperature of the body is maintained constant.

9. The amount of heat, dH, absorbed during an elementary transformation, is composed of two parts: 1st, the variation of the actual amount of heat contained in the substance, which amount equals $\frac{1}{\text{E}}\varphi s$; 2d, the heat absorbed by the total work done during the elementary transformation, which is $d\text{R} = \varphi ds$.

Hence: $$\text{E}d\text{H} = d(\varphi s) + \varphi ds$$

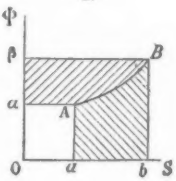
Or: $$\text{E}d\text{H} = sd\varphi + 2\varphi ds \qquad (9)$$

Whence, for a finite transformation:

$$\text{EH} = \int_a^\beta sd\varphi + 2\int_a^b \varphi ds$$

We see from this equation, that the amount of heat necessary to let the substance describe a certain path AB (fig. 2) is proportional to the area $Aa_1\beta B$ plus twice the area $AabB$ (both of these areas being determined by the path AB).

2.



When a substance undergoes a transformation without transmission of heat, the path described is called an "*adiabatic*" or "*isentropic line.*" This path is determined by the condition:

$$d\text{H} = 0$$

Or $$sd\varphi + 2\varphi ds = 0$$

Whence by integration:

$$\varphi s^2 = \text{constant}$$

Such is the general equation to the adiabatic lines for any substance. These lines are of the third degree and belong to the hyperbolic species.

10. *Clausius' Theorem.*—Equation (9) can be written:

$$\mathrm{E}d\mathrm{H} = \varphi s\left(\frac{d\varphi}{\varphi} + 2\frac{ds}{s}\right)$$

Or again, by the aid of the relation: $\varphi s = \mathrm{KTE}$:

$$\frac{d\mathrm{H}}{\mathrm{T}} = \mathrm{K}\left(\frac{d\varphi}{\varphi} + 2\frac{ds}{s}\right)$$

Whence, for a finite transformation:

$$\int_{\mathrm{A}}^{\mathrm{B}} \frac{d\mathrm{H}}{\mathrm{T}} = \mathrm{K}(\log \varphi_{\mathrm{B}} s_{\mathrm{B}}^2 - \log \varphi_{\mathrm{A}} s_{\mathrm{A}}^2)$$

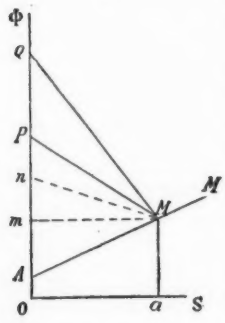
When the path is a closed cycle, A = B; in this case:

$$\int \frac{d\mathrm{H}}{\mathrm{T}} = 0$$

Graphical representation of the Specific Heat.

11. Let M be the initial state of the substance (fig. 3); and suppose an elementary path MM' to be described in a certain direction. Amongst the infinite number of directions around M, some are of special interest:

3.



1st. The direction of the isothermal, whose equation is: $\varphi s = \varphi_0 s_0$; φ_0 and s_0 denoting the coördinates of point M. This equation has been derived from the condition: T = constant or $d\mathrm{T} = 0$.

2d. The direction of the adiabatic, whose equation is: $\varphi s^2 = \varphi_0 s^2_0$, derived from the condition: $d\mathrm{H} = 0$.

All the other directions of special interest are found in the same way by equating to zero one of the differential quantities entering the equations. Thus, we shall obtain:

3d. The direction and equation of the path described by the substance when its volume is maintained constant, by putting $d\mathrm{V} = 0$ or V = constant; substituting to V its value in terms of φ and s as given in equations (8), gives for the equation to the curve of constant volume passing through M:

$$g(\varphi, s) = g(\varphi_0, s_0)$$

4th. The equation of the path described when the pressure is maintained constant, by putting $dP = 0$ or $P =$ constant, or again by the same substitution :

$$f(\varphi, s) = f(\varphi_0, s_0)$$

5th. The equation to the curve of constant symbolical volume; since $s = \pi a^2$, the amplitude a is constant when s is constant, hence this curve can be defined as the path described by the substance when the amplitude of the vibratory motion is maintained constant and its equation is found by putting $ds = 0$, whence :

$$s = s_0$$

which is the equation to a straight line parallel to the axis of φ.

6th. The curve of constant symbolical pressure, which, since $\varphi = \frac{\pi}{i^2}$, can be defined as the path described when the period of the vibratory motion of the heat is maintained constant. The equation of this curve is derived from the condition : $d\varphi = 0$, whence:

$$\varphi = \varphi_0$$

which represents a straight line parallel to the axis of s.

7th. The path along which the total work $d\mathrm{R} = 0$, or $\varphi ds = 0$, or :

$$s = s_0$$

which is the same as the equation to the curve of constant symbolical volume.

8th. The path along which the external work $d\tau = 0$ or $\mathrm{P}d\mathrm{V} = 0$ or:

$$\mathrm{V} = \text{constant}$$

We see here again that the symbolical volume is in the same relation with the total work as is the ordinary volume with the external work.

We might also remark that since $d\mathrm{R} = 0$ along the curve of constant symbolical volume, this curve may be defined as representing a transformation in which all the heat furnished is used in raising the temperature of the substance (or increasing its heat-energy), no part of it being transformed into work.

9th. The path along which the internal work, $d\mathrm{I} = 0$ or: $d\mathrm{R} - dT = 0$ or again: $\varphi ds - \mathrm{P}d\mathrm{V} = \mathrm{O}$. Expressing P and V in terms of φ and s, with the aid of equations (8):

$$\varphi ds - f(\varphi, s)d[g(\varphi, s)] = 0$$

which will furnish by integration the equation of the required curve.

Such are the principal curves passing through any point M. When M describes an elementary path MM', the direction of

MM' determines the value of the quotient $\frac{d\varphi}{ds}$. The heat absorbed in this elementary transformation is given by:

$$\mathrm{E}d\mathrm{H} = sd\varphi + 2\varphi ds$$

and the corresponding variation of temperature, by :

$$\mathrm{KE}d\mathrm{T} = sd\varphi + \varphi ds$$

Dividing member to member, and putting $\frac{d\mathrm{H}}{d\mathrm{T}} = \gamma$ for abreviation, we have:

$$\frac{\gamma}{\mathrm{K}} = \frac{sd\varphi + 2\varphi ds}{sd\varphi + \varphi ds} \qquad (10)$$

The quotient $\gamma = \frac{d\mathrm{H}}{d\mathrm{T}}$ may be called the specific heat of the substance at the state M and for the direction MM' (since γ varies with the value of $\frac{d\varphi}{ds}$, i. e., the direction of MM').

When $\frac{d\varphi}{ds}$ has such a value that the direction MM' coincides with that of the tangent to the curve of constant volume, the corresponding value of γ is evidently the "*specific heat at constant volume*" at the state M, which specific heat we shall denote by the letter c.

In the same way, when $\frac{d\varphi}{ds}$ has such a value that MM' coincides with the tangent to the curve of constant pressure, the corresponding value of γ is by definition the "*specific heat at constant pressure*" of the substance at the state M; this specific heat will be denoted by the letter C.

The value of γ corresponding to a direction parallel to the axis of φ may be called for the same reason: the "*specific heat at constant symbolical volume;*" and that corresponding to a direction parallel to the axis of s: the "*specific heat at constant symbolical pressure.*"

Now, equation (10) can be written:

$$\frac{\gamma}{\mathrm{K}} = \frac{s\frac{d\varphi}{ds} + 2\varphi}{s\frac{d\varphi}{ds} + \varphi}$$

Let us produce MM' until it intersects the axis of φ at point A, and measure on this axis three equal lengths: om, mP and PQ, each one of them equal to the ordinate φ of point M; then, we shall have:

$$\overline{m\mathrm{A}} = s\frac{d\varphi}{ds} \quad \overline{m\mathrm{Q}} = 2\varphi \text{ and } \overline{m\mathrm{P}} = \varphi$$

Whence, by substitution:

$$\frac{\gamma}{K} = \frac{mA + \overline{mQ}}{mA + mP} = \frac{\overline{AQ}}{\overline{AP}}$$

Thus we see that the specific heat corresponding to any direction $\overline{MA}$ is to the absolute specific heat of the substance as the distances from point A (determined by the direction $\overline{MA}$ itself) to the stationary points P and Q.

The specific heats c and C can then be easily obtained graphically by tracing the tangents at point M to the curve of constant volume and to the curve of constant pressure; or, if desired, we can also make use of the specific heats to find these tangents. Let us study now the variation of the specific heat γ, when the direction of the element MM' changes, by revolving around point M.

1st. Suppose that MM' or MA be at first parallel to the axis of φ; point A is then removed to an infinite distance in the negative direction of the axis of φ and we have: $\frac{\gamma}{K} = \frac{\overline{AQ}}{\overline{AP}} = 1$ or $\gamma = K$. In other words, the specific heat at constant symbolical volume is equal to the absolute specific heat of the substance, and therefore does not depend on its physical state.

2d. When $\overline{MA}$, by revolving ninety degrees around M, coincides with $\overline{Mm}$ we have: $\frac{\gamma}{K} = \frac{\overline{AQ}}{\overline{AP}} = \frac{\overline{mQ}}{mP} = 2$, or $\gamma = 2K$, i. e.: the specific heat at constant symbolical pressure is equal to twice the absolute specific heat of the substance and remains therefore also constant, when the state of the substance changes.

3d. When $\overline{MA}$ has reached the position $\overline{Mn}$ (n being the center of $\overline{mP}$), $\frac{\gamma}{K} = \frac{\overline{AQ}}{\overline{AP}} = \frac{\overline{nQ}}{nP} = 3$, or $\gamma = 3K$.

This value is of some interest, since the specific heat at constant volume of certain solid substances has been found to be equal to three times their absolute specific heat; hence, for those substances, $\overline{Mn}$ is the tangent at M to the curve of constant volume.

4th. When $\overline{MA}$ coincides with MP:

$$\frac{\gamma}{K} = \frac{\overline{AQ}}{\overline{AP}} = \frac{\overline{PQ}}{0} = \infty$$

Whence: $\gamma = \frac{dH}{dT} = \infty$ or $dT = 0$ or again $T = \text{constant}$. This

shows that MP is the tangent to the isothermal passing through M.

5th. From P to Q, the quotient $\frac{\overline{AQ}}{\overline{AP}}$ becomes negative and is decreasing, i. e., dH and dT have opposite signs.

6th. When MA coincides with MQ:

$$\frac{\gamma}{K} = \frac{\overline{AQ}}{\overline{AP}} = \frac{0}{\overline{PQ}} = 0$$

whence: $\gamma = \frac{dH}{dT} = 0$ or $dH = 0$. Hence $\overline{MQ}$ is the tangent to the adiabatic passing through M.

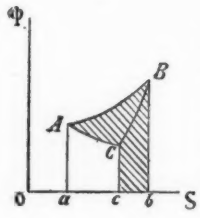
7th. Above point Q, $\frac{\overline{AQ}}{\overline{AP}}$ is again positive and increases from zero (at Q) to one (at infinity).

Thus the tangents MP and MQ to the isothermal and to the adiabatic, divide the space around point M into four regions. When the path described by the substance passes from one of these regions into another, a change of sign occurs either in dH or in dT; this change taking place for dH on the adiabatic, and for dT on the isothermal.

The points P and Q remain stationary, not only when $\overline{MA}$ revolves around point M but also when point M itself moves on a parallel to the axis of s (since the position of P and Q depends only on the ordinate φ of point M). Hence if we trace the tangents to the isothermal and to the adiabatic at each point of this parallel, these tangents will form two pencils converging respectively at P and at Q. For the same reason, if we draw the tangents to the curve of constant pressure or of constant volume, at two different points on this parallel, these two tangents shall intersect each other on the axis of φ (provided however that the two points be sufficiently near each other so that the specific heats can be regarded as having practically the same value for both points).

12. We have seen that the area $AabB$ (fig. 4) is equal to the total work, R, absorbed during the transformation AB. In order to determine graphically the value of the external work T and that of the internal work I, trace the curve of constant volume AC and also the curve BC along which the internal work $dI = 0$. These curves intersect at C and the area $AabB$ is now divided into two parts: the hatched part $ACcbB$ being equal to the external work T, while the remaining part $AacC$ is equal to the internal work, absorbed by the substance when describing the path AB.

4.



The demonstration is based upon the fact that the internal work depends only upon the initial and final states of the body and not upon the path described, so that the internal work along AB is equal to the internal work along ACB; but, by hypothesis, the internal work along CB equals zero; hence, the internal work from A to B is equal to the internal work from A to C. On the other hand, the total work from A to C is given by the area $AacC$; but the external work from A to C is equal to zero, since the volume remains constant along the path AC; hence the area $AacC$ is also equal to the internal work from A to C, which is the same as the internal work from A to B, as just seen.

The area $AacC$ giving the internal work I along the path AB, the remaining area $ACcbB$ must be equal to the external work T along the same path. Since the external work can be easily computed in most cases, this demonstration gives us also a graphical method for tracing, through any point B, the curve defined by the condition $d\mathrm{I} = 0$: trace any path BA through B, compute the external work from B to A and subtract it from area $AabB$; the result is the internal work from B to A; trace through A the curve of constant volume AC and an ordinate cC cutting off an area $AacC$ equal to the internal work previously computed. The point C thus obtained is a point of the required curve. Other points can be found in the same way, by moving point A on the curve BA. Since the path BA is arbitrary, it must be chosen so that the external work can be easily computed; for instance, if BA be the curve of constant pressure passing through B, then the external work:

$$\int_{\mathrm{B}}^{\mathrm{A}} \mathrm{P}d\mathrm{V} = \mathrm{P}\int_{\mathrm{B}}^{\mathrm{A}} d\mathrm{V} = \mathrm{P}(\mathrm{V}_{\mathrm{A}} - \mathrm{V}_{\mathrm{B}})$$

Remark: When the path described by point M is a closed cycle, the area enclosed by it is equal to the total work absorbed by the substance; but, since the final state is the same as the initial state, the internal work is equal to zero; hence said area is also equal to the *external* work done by the substance, as this is the case in Clapeyron's graphical representation where P and V are taken as coördinates; this result can be expressed by the equation:

$$\int q d s = \int \mathrm{P}d\mathrm{V}$$

which applies only to a closed cycle. We shall soon express the same equation under a different form.

Graphical representation of the various physical coefficients.

13. When a surface is defined by three equations giving the value of the coördinates P, V, T of any one of its points, in terms of two auxiliary variables φ and s, it can be divided into an infinite number of infinitely small parallelograms by two systems of curves, obtained respectively by putting φ = constant and s = constant in the three equations to the surface. The area $d\omega$ of any one of these elementary parallelograms is given by the formula:

$$d\omega = (A^2 + B^2 + C^2)^{\frac{1}{2}} d\varphi ds$$

in which A, B, C have the following values:

$$\begin{cases} \pm A = \dfrac{dV}{d\varphi}\dfrac{dT}{ds} - \dfrac{dT}{d\varphi}\dfrac{dV}{ds} \\ \pm B = \dfrac{dT}{d\varphi}\dfrac{dP}{ds} - \dfrac{dP}{d\varphi}\dfrac{dT}{ds} \\ \pm C = \dfrac{dP}{d\varphi}\dfrac{dV}{ds} - \dfrac{dV}{d\varphi}\dfrac{dP}{ds} \end{cases}$$

The projection of the element $d\omega$ on the plane POV is equal to: $\pm C d\varphi ds$; and the area of this projection is equal, according to the fundamental property of Clapeyron's graphical representation, to the external work done by the substance when describing a path corresponding to the outline of the parallelogram $d\omega$. On the other hand, when φ and s are taken as coördinates, the element $d\omega$ is represented by a small rectangle, whose sides are respectively $d\varphi$ and ds, since this element is cut off by the curves: φ = constant and s = constant; the area of this rectangle is: $d\varphi . ds$ and is equal, according to the last remark (§ 12), to the external work done by the substance when describing the outline of the same element $d\omega$, hence:

$$\pm C d\varphi ds = d\varphi ds$$

Or:

$$C = \pm 1$$

Replacing C by its value, we find a new condition to which the functions f and g (equations 8) are submitted, i. e.:

$$\frac{df}{d\varphi}\frac{dg}{ds} - \frac{dg}{d\varphi}\frac{df}{ds} = \pm 1$$

This condition will be utilized in the determination of the functions f and g, as it is a more convenient form of the condition found above: $\int \varphi ds = \int P dv$, for any closed cycle.

By the aid of equation $T = \dfrac{\varphi s}{KE}$, we have:

$$\frac{d\mathrm{T}}{d\varphi} = \frac{s}{\mathrm{KE}} \text{ and } \frac{d\mathrm{T}}{ds} = \frac{\varphi}{\mathrm{KE}}$$

Hence:

$$\begin{cases} \mathrm{A} = \pm\dfrac{1}{\mathrm{KE}}\left(\varphi\dfrac{d\mathrm{V}}{d\varphi} - s\dfrac{d\mathrm{V}}{ds}\right) \\ \mathrm{B} = \pm\dfrac{1}{\mathrm{KE}}\left(s\dfrac{d\mathrm{P}}{ds} - \varphi\dfrac{d\mathrm{P}}{d\varphi}\right) \\ \mathrm{C} = \pm 1 \end{cases} \qquad (11)$$

These formulæ give the value of A, B, C corresponding to any physical state of the substance defined by the coördinates φ and s.

If we now express each one of the physical coefficients in terms of A, B, C we shall be able to compute the value of these coefficients for any state of the substance, by means of equations (8), since A, B, C are depending upon equations (8) by means of equations (11).

When the thermodynamic surface is given under the ordinary form $\mathrm{T} = f(\mathrm{P}, \mathrm{V})$, it is always possible to introduce two auxiliary variables, by putting:

$$\begin{cases} \mathrm{P} = u \\ \mathrm{V} = v \\ \mathrm{T} = f(u, v) \end{cases}$$

whence:

Under this form, the area $d\varepsilon$ of the elementary parallelograms cut off by the curves $u =$ constant and $v =$ constant, is:

$$d\varepsilon = (\mathrm{A}_1^2 + \mathrm{B}_1^2 + \mathrm{C}_1^2)^{1/2} du dv$$

in which:

$$\mathrm{A}_1 = \pm\frac{d\mathrm{T}}{d\mathrm{P}} \quad \mathrm{B}_1 = \pm\frac{d\mathrm{T}}{d\mathrm{V}} \quad \mathrm{C}_1 = \pm 1$$

The element $d\varepsilon$ is not the same as the element $d\omega$, but it has the same tangent plane. If we measure off on the normal to the surface a length:

$$\mathrm{N} = (\mathrm{A}^2 + \mathrm{B}^2 + \mathrm{C}^2)^{1/2}$$

the components of N parallel to the axes of coördinates shall be respectively: A, B and C. Now, since the normal and the axes of coördinates are the same for both elements and since the components C and C_1 are equal ($\mathrm{C} = \pm 1 = \mathrm{C}_1$), the other two components must be equal to each other, thus:

$$\mathrm{A} = \mathrm{A}_1 = \pm\frac{d\mathrm{T}}{d\mathrm{P}}$$

$$\mathrm{B} = \mathrm{B}_1 = \pm\frac{d\mathrm{T}}{d\mathrm{V}}$$

In other words:

$$\frac{dP}{dT} = \pm \frac{1}{A},\ V \text{ being constant}$$

$$\frac{dV}{dT} = \pm \frac{1}{B},\ P \text{ being constant.}$$

The various physical coefficients can now be expressed in terms of A and B:

1st. *Coefficient of dilatation at constant pressure* (α).—We have by definition:

$$\alpha = \frac{1}{V}\frac{dV}{dT},\ P \text{ being constant}$$

Hence: $$\alpha = \pm \frac{1}{VB}$$

2d. *Coefficient of dilatation at constant volume* (β).—Again, by definition:

$$\beta = \frac{1}{P}\frac{dP}{dT},\ V \text{ being constant.}$$

Whence: $$\beta = \pm \frac{1}{PA}$$

3d. *Coefficient of compressibility at constant temperature* (μ).—By definition:

$$\mu = -\frac{1}{V}\frac{dV}{dP},\ T \text{ being constant.}$$

Whence: $$\mu = \mp \frac{1}{V}\frac{A}{B}$$

4th. *Specific heat at constant pressure* (C).—We have found: $\frac{C}{K} = \frac{sd\varphi + 2\varphi ds}{sd\varphi + \varphi ds}$, the quotient $\frac{d\varphi}{ds}$ being here defined by the condition: P = constant, or:

$$dP = \frac{dP}{d\varphi}d\varphi + \frac{dP}{ds}ds = 0$$

By combining these two equations, we shall obtain:

$$C - K = \pm \frac{\varphi}{EB}\frac{dP}{d\varphi}$$

5th. *Specific heat at constant volume* (c).—Similar equations lead to the similar result:

$$c - K = \pm \frac{\varphi}{EA}\frac{dV}{d\varphi}$$

Remark: Subtracting this equation from the preceding one, it gives:

$$C - c = \pm \frac{T}{E}\frac{1}{AB}$$

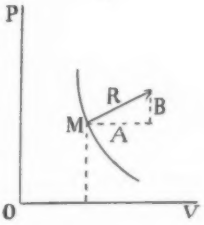
6th. *Latent heat of dilatation* (l):

$$l = \frac{T}{E}\frac{dP}{dT} = \pm \frac{T}{EA}$$

In short, all the different physical coefficients can be expressed very simply in terms of A and B, so that if the value of A and B can be obtained by a graphical method, these coefficients may also be regarded as known graphically.

A and B are the components of N parallel to the axes OV and OP, i. e.: parallel to the axes of coördinates used in Clapeyron's graphical method. Hence if M represents the state of the substance (fig. 5), defined by its volume and its pressure, the component A will be parallel to OV, while B will be parallel to OP and the resultant R will be the projection of N on the plane POV.

5.



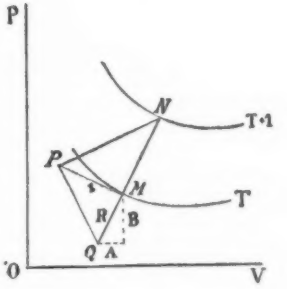
Now, if R be known both in length and direction, the components A and B (and therefore all the physical coefficients) can be obtained graphically, so that it will not be necessary to compute A and B from the equations to the thermodynamic surface. The projecting plane of N is perpendicular to the contour lines of the thermodynamic surface (i. e., supposing that the plane POV is horizontal). These contour lines are precisely the isothermal curves, hence the direction of R can be obtained graphically by the condition that it be perpendicular to the isothermal MT passing through M.

On the other hand, the length of R is determined by the fact that the vertical component C is equal to the unit.

The altitude of point M in space is equal to the temperature T; hence, since $C = 1$, the altitude of the other extremity of N is equal to $T+1$; in other words, this extremity is the point where the normal to the thermodynamic surface intersects the horizontal plane of the isothermal $T+1$, so that it can be obtained graphically by a simple construction of descriptive geometry, as follows:

Draw through M the normal to the isothermal MT (fig. 6) and let N denote the point where this normal intersects the isothermal $T+1$; draw MP tangent to the isothermal and measure off $MP=1$; join N to P; draw at P a perpendicular to NP and let Q denote the point where it intersects the normal MN produced; then MQ will be equal to R both in length and direction.

6.



The isothermal $T+1$ can be replaced by the isothermal $T+n$, pro-

vided that the result be divided by n, and also that the portion MN of the normal be short enough to be practically straight, notwithstanding the curvature of the surface. R being obtained graphically, the components A and B are readily obtained by drawing parallels to the axes of coördinates through the points M and Q.

This graphical determination of R, does not apply when φ and s are chosen as coördinates: in this case A and B have to be computed by the aid of equations (11); but, when this has been done, A and B can still be considered as two magnitudes parallel to the axes $o\varphi$ and os and they can be combined into a single one R having a definite direction, so that in any case, all the physical coefficients referring to a particular state of the substance can be represented graphically by a single magnitude of a certain length, drawn in a proper direction.

Research of the thermodynamic function.

14. The thermodynamic function of a given substance may be known under the usual form: $F(P, V, T) = 0$, or it may not be known at all, as is the case for most substances.

In the first case, the thermodynamic function can be expressed in terms of φ and s by the aid of the equations:

$$\begin{cases} dH = c dT + l dV \\ dH = C dT + h dP \\ \dfrac{dH}{T} = K\left(\dfrac{d\varphi}{\varphi} + 2\dfrac{ds}{s}\right) \\ \dfrac{dT}{T} = \dfrac{d\varphi}{\varphi} + \dfrac{ds}{s} \end{cases}$$

By combining these equations, they can be written:

$$\begin{cases} \dfrac{l}{T} dV + (c - K)\dfrac{dT}{T} = K\dfrac{ds}{s} \\ \dfrac{l}{T} dV + (c - 2K)\dfrac{dT}{T} = -K\dfrac{d\varphi}{\varphi} \end{cases} \qquad (12)$$

$$\begin{cases} \dfrac{h}{T} dP + (C - K)\dfrac{dT}{T} = K\dfrac{ds}{s} \\ \dfrac{h}{T} dP + (C - 2K)\dfrac{dT}{T} = -K\dfrac{d\varphi}{\varphi} \end{cases}$$

Let us take for instance the case of the perfect gases, whose thermodynamic function is $PV = RT$, R being a constant.

If in the third of the equations to the thermodynamic surface, which is $\varphi s = KTE$ for any substance, φ and s be replaced by their value in terms of P and V, obtained from the first

two equations, the result will be the thermodynamic function under the usual form: so that we might say that the thermodynamic function of any substance expresses the fact that the product of the symbolical pressure by the symbolical volume is proportional to the absolute temperature.

But the thermodynamic function of perfect gases expresses also that the product of the pressure by the volume is proportional to the absolute temperature. Hence, for perfect gases, the symbolical pressures and volumes can be respectively replaced by the ordinary pressures and volumes; and since there is the same relation between the total work and the symbolical pressure and volume, as there is between the external work and the ordinary pressure and volume, it follows that for perfect gases the total work is equivalent to the external work, in other words there is no internal work in perfect gases. So that, for these gases:

$$\int \varphi ds = \int \mathrm{P} d\mathrm{V}$$

even when the cycle is not closed.

For the same reason, the curves of constant volume must be the same as the curves of constant symbolical volume, i. e., they must be straight lines parallel to the axis of φ, for if s be constant, $ds = 0$ or: $\varphi ds = \mathrm{P} d\mathrm{V} = 0$, hence $d\mathrm{V} = 0$ or $\mathrm{V} =$ constant. And according to what has been said in the graphical determination of specific heats, it follows also that the specific heat at constant volume of a perfect gas is equal to its absolute specific heat: $c = K$. All these results are well known, but as they have been established only by experiment, we have tried to show how they are direct consequences of the laws of thermodynamics.

Now that $c = K$ for perfect gases, equation (12) reduces to:

$$\frac{l}{\mathrm{T}} d\mathrm{V} = \mathrm{K} \frac{ds}{s}$$

On the other hand:

$$\frac{l}{\mathrm{T}} = \frac{1}{\mathrm{E}} \frac{d\mathrm{P}}{d\mathrm{T}} = \frac{\mathrm{R}}{\mathrm{EV}}$$

Hence, by substitution:

$$\frac{\mathrm{R}}{\mathrm{E}} \frac{d\mathrm{V}}{\mathrm{V}} = \mathrm{K} \frac{ds}{s}$$

and by integration:

$$\mathrm{V} = \mathrm{M} s^{\frac{\mathrm{KE}}{\mathrm{R}}}$$

M being an arbitrary constant. We have also, for any substance:

$$T = \frac{\varphi s}{KE}$$

Substituting these values of V and T in the thermodynamic function: $PV = RT$, it gives:

$$P = \frac{R}{MKE} \varphi s^{1-\frac{KE}{R}}$$

Finally, if we put $\frac{KE}{R} = a$ for abreviation, the thermodynamic function of perfect gases, expressed in terms of φ and s, is:

$$\begin{cases} P = \dfrac{\varphi s^{1-a}}{aM} \\ V = Ms^a \\ T = \dfrac{\varphi s}{aR} \end{cases}$$

Remarks: By differentiating the second of these equations and multiplying the result by the first one, we see that $PdV = \varphi ds$ as stated above. The equation $V = Ms^a$ shows that the volume of a perfect gas happens to be independent of φ, i. e., of the period of the vibratory motion. In other words, the amplitude a keeps the same value as long as the volume remains the same (V being constant when s or a is constant). This is not true of other substances, as in the general case, the expression of V involves φ as well as s($V = g(\varphi, s)$). This example shows how the thermodynamic function can be expressed in terms of φ and s, when it is already known under the usual form: $F(P, V, T) = 0$. In general, this problem consists in transforming the given equation $F(P, V, T) = 0$ into three others giving the value of P, V and T in terms of two auxiliary variables φ and s, which must satisfy the conditions found above:

$$\varphi s = KTE$$

$$\frac{dP}{d\varphi}\frac{dV}{ds} - \frac{dV}{d\varphi}\frac{dP}{ds} = \pm 1$$

Let us examine now the case in which the thermodynamic function is entirely unknown. In Clapeyron's graphical process, the path described by a substance can always be traced on the paper by measuring directly the volume and the pressure for a sufficient number of points along the path, whether the thermodynamic function be known or not. When φ and s are chosen as coördinates the position of the point corresponding to any physical state of the substance can also be determined directly from experimental data.

Let us suppose first that the specific heat at constant volume of the substance is constant, as is the case for a certain number of bodies, and let us find what would be in this hypothesis, the general equation of the curves of constant volume. By putting V = constant or $dV = 0$ in equation (12) and replacing $\frac{dT}{T}$ by: $\frac{d\varphi}{\varphi} + \frac{ds}{s}$, we shall obtain the differential equation:

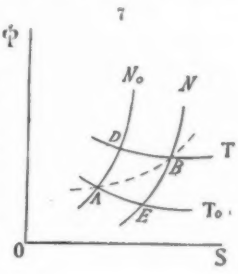
$$(c-K)\frac{d\varphi}{\varphi} + (c-2K)\frac{ds}{s} = 0$$

Integrating and denoting the arbitrary constant by N, the general equation to the curves of constant volume is:

$$\varphi^{c-K} s^{c-2K} = N$$

On the other hand: $\varphi s = KTE$ is the general equation of the isothermals. The value of the constant N depends only upon the volume V, so that as soon as the experimental data V and T are given, these equations will furnish the corresponding value of φ and s, provided however that the value of the constant N be known for any given value of the volume V.

This determination can be made as follows: let V_0 and T_0 be the experimental data corresponding to the initial state of the substance. The point representing this state, is on the isothermal: $\varphi s = KT_0E$; but it can be chosen anywhere on this isothermal, because the three equations to the thermodynamic surface involve always an arbitrary constant, as seen in the case of a perfect gas. Let A be the chosen point (fig. 7); φ_0 and s_0, its coördinates; then the value of the constant N, corresponding to the initial value of the volume, is given by the equation:



$$N_0 = \varphi_0^{c-K} s_0^{c-2K}$$

Let the substance describe any path, such as AB and let V and T be the experimental data corresponding to any point, such as B; the coördinates φ and s of this point will be determined by the intersection of the two curves:

$$\left.\begin{array}{ll} DBT: & \varphi s = KTE \\ \text{and} \quad EBN: & \varphi^{c-K} s^{c-2K} = N \end{array}\right\}$$

N being given a value corresponding to that of the volume V; to find this value of N, it must be noticed that the isothermals AET_0, DBT and the curves of constant volume ADN_0, EBN, form a curved quadrilateral $AEBD$, whose area is equal to the external work done by the substance, if it were made to

describe the closed path $AEBD$. The amount of this work can be easily computed from the experimental data and the area of the quadrilateral can also be obtained analytically from the equations of its sides:

$$\left.\begin{array}{ll} AE: & \varphi s = \mathrm{KT_0E} \\ DB: & \varphi s = \mathrm{KTE} \\ AD: & \varphi^{c-\mathrm{K}} s^{c-2\mathrm{K}} = \mathrm{N_0} \\ EB: & \varphi^{c-\mathrm{K}} s^{c-2\mathrm{K}} = \mathrm{N} \end{array}\right\}$$

Since the last equation involves the unknown constant N, the area of the quadrilateral will be obtained in terms of N, and by equating said area to the external work previously computed in terms of V and T, we shall obtain an equation giving the unknown constant N in terms of the experimental data.

In short N can be considered as known, as soon as the volume V is given, so that the two equations:

$$\left\{\begin{array}{l} \varphi^{c-\mathrm{K}} s^{c-2\mathrm{K}} = \mathrm{N} \\ \varphi s = \mathrm{KTE} \end{array}\right.$$

can be regarded as giving the value of the coördinates φ and s in terms of the experimental data V and T. These equations enable us to find the position of the point corresponding to any given physical state of the substance, without having to know its thermodynamic function, provided only that its specific heat be known.

The equation to the curves of constant volume can also be made use of, for the determination of the thermodynamic function in terms of φ and s; for, since we know how to find the value of the constant N corresponding to any given value of the volume V, it may be possible to express N in terms of V by an empirical function: $\mathrm{N} = \psi(\mathrm{V})$. In this case, the general equation to the curves of constant volume can be written:

$$\psi(\mathrm{V}) = \varphi^{c-\mathrm{K}} s^{c-2\mathrm{K}}$$

This equation gives V in terms of φ and s; hence it is one of the three equations involved in the thermodynamic function.

All that has been said in this paragraph concerning the volume V, applies also to the pressure P, provided the specific heat at constant volume c, be replaced by the specific heat at constant pressure C. The general equations to the curves of constant pressure would be:

$$\chi(\mathrm{P}) = \varphi^{\mathrm{C}-\mathrm{K}} s^{\mathrm{C}-2\mathrm{K}}$$

($\chi(P)$ being a function obtained empirically), and the complete thermodynamic function of the substance would then be:

$$\left\{\begin{array}{l} \chi(\mathrm{P}) = \varphi^{\mathrm{C}-\mathrm{K}} s^{\mathrm{C}-2\mathrm{K}} \\ \psi(\mathrm{V}) = \varphi^{c-\mathrm{K}} s^{c-2\mathrm{K}} \\ \mathrm{KTE} = \varphi s \end{array}\right.$$

This form of the thermodynamic function applies only to the case in which the specific heats are constant.

In the general case, the specific heats vary with the physical state of the substance; but as this variation is always comparatively small, the sheet of paper can be divided into a sufficient number of regions, so that either one of the specific heats of the substance may be regarded as constant inside of any one of these regions. Then the equations given above will hold true, provided that the specific heat be given a special value for each region.

Another method consists in finding first the value of the specific heat in terms of the volume and the temperature; since the variation of c is slow, it will be sufficient to take only in consideration the first and second powers of V and T, so that it can always be assumed that:

$$c = a + bV + kT + dT^2 + eVT + fV^2$$

a, b, k, etc., being constant coefficients determined empirically.

The differential equation to the curves of constant volume being still:

$$(c - K)\frac{dT}{T} = K\frac{ds}{s}$$

or, by replacing c by its value:

$$\left(\frac{a+bV+fV^2-K}{T} + k + eV + dT\right)dT = K\frac{ds}{s}$$

We shall obtain by integration, V being constant:

$$(a+bV+fV^2-K)\log T + (k+eV)T + \tfrac{1}{2}dT^2 = K\log s + \text{constant}.$$

Replacing T by its value $\frac{\varphi s}{KE}$, the result will be the equation of the curves of constant volume, involving only one unknown constant, which can be determined by the method given above.

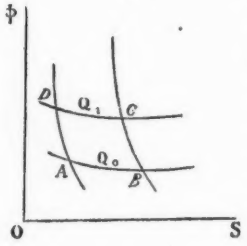
Conclusion.

15. The object we have been trying to fulfill in this short study, was to determine which is the best system of coördinates to adopt in graphical representations of thermodynamical phenomena. Evidently, the best system is the one in which the value of each variable depending upon the phenomenon can be obtained graphically; hence, each one of these variables must be represented by a geometrical magnitude depending only upon the form and position of the path described by the substance with respect to said system of coördinates.

The system of coördinates φ, s seems to possess this property to a higher degree than the system P, V, which is usually adopted and known as Clapeyron's system.

It is true that in the latter system, the location of the point representing any state of the substance, is obtained directly from the experimental data P and V, while in the system φ, s the point has to be located by a more or less indirect way; but when the path of the substance has been traced in Clapeyron's system, it does not give any information relating to the phenomena (except the value of the external work, but this work can usually be computed without difficulty); if it is desired to know more about the transformation, it is necessary to trace first all the isothermals and adiabatics; this is a long work, these curves being irregular curves in the system P, V. On the contrary, when φ and s are chosen as coördinates, the path itself is sufficient to give any information concerning the phenomenon, because the various physical variables depending upon the transformation are given by geometrical magnitudes determined by the path itself, and also because the isothermals and adiabatics, instead of being irregular, are geometrical curves whose equations are known, being the same for any substance; so that the curves: $\varphi s =$ constant, and: $\varphi s^2 =$ constant, need to be traced but once for all; they can be reproduced as many times as necessary by the blue-print process or by any other printing process.

8.



The use of the coördinates φ and s simplifies also the demonstration of most of the theorems of thermodynamics; let us take, as an example, Carnot's theorem.

If a substance describes a closed path formed by two adiabatics AD, CB (fig. 8) and by two isothermals DC, BA, and if the amount of heat absorbed and abandoned by the substance be denoted respectively by H_1 and H_0, the expression $\frac{H_1 - H_0}{H_1}$, which is called the *economical coefficient* of the cycle, is independent of the nature of the substance, and is equal to $\frac{T_1 - T_0}{T_1}$, T_0 and T_1 being the temperatures corresponding to the isothermals.

Since there is no transmission of heat along the adiabatics AD and CB, the amounts of heat H_1 and H_0 correspond respectively to the isothermals DC and BA. We have seen that:

$$EH = \int s d\varphi + 2\int \varphi ds$$

the equation to the isothermals being: $\varphi s = \mathrm{KTE}$, the integration gives:

$$\mathrm{EH} = \mathrm{KTE}(\log \varphi + 2 \log s)$$

Hence:

$$\mathrm{EH_1} = \mathrm{KT_1E}(\log \varphi_{\mathrm{D}} s^2_{\mathrm{D}} - \log \varphi_{\mathrm{C}} s^2_{\mathrm{C}})$$
$$-\mathrm{EH_0} = \mathrm{KT_0E}(\log \varphi_{\mathrm{B}} s^2_{\mathrm{B}} - \log \varphi_{\mathrm{A}} s^2_{\mathrm{A}})$$

But the equation to the adiabatics being: $\varphi s^2 = \text{constant}$, we have also: $\varphi_{\mathrm{C}} s^2_{\mathrm{C}} = \varphi_{\mathrm{B}} s^2_{\mathrm{B}}$ and $\varphi_{\mathrm{D}} s^2_{\mathrm{D}} = \varphi_{\mathrm{A}} s^2_{\mathrm{A}}$, so that by dividing:

$$\frac{\mathrm{H_1}}{\mathrm{T_1}} = \frac{\mathrm{H_0}}{\mathrm{T_0}}$$

which can be written:

$$\frac{\mathrm{H_1} - \mathrm{H_0}}{\mathrm{H_1}} = \frac{\mathrm{T_1} - \mathrm{T_0}}{\mathrm{T_1}}$$

Thus, the demonstration of the theorem consists simply in the analytical measurement of areas, and the reason of it is that the heat of transformation *H*, which is the unknown quantity, is represented graphically by areas. This is not the case in the usual graphical methods, and Carnot's Theorem can only be demonstrated indirectly, by showing first that the economical coefficient of Carnot's cycle is independent of the nature of the substance, then computing its value for a perfect gas. One can object to this method that a perfect gas is only a theoretical substance, having no real existence.

The use of the coördinates ϕ and s, does not imply any hypothesis upon the nature of the motion constituting heat, as ϕ and s can be regarded simply as two auxiliary variables, without attributing to them any special significance.

Another advantage of expressing the thermodynamic function in terms of ϕ and s, is that each one of the specific heats of the substance can be obtained separately from said function, while the ordinary form of the thermodynamic function $\mathrm{F(P, V, T)} = 0$ furnishes only the difference between the two specific heats $(\mathrm{C} - c)$. Finally, if heat is really a vibratory motion of the particles of matter, the state of this vibration at any time is obtained from the experimental data, by solving the thermodynamic function with respect to ϕ and s.

ART. IV.—*On the Application of the Schroeder-Le Chatelier Law of Solubility to Solutions of Salts in Organic Liquids;* by C. E. LINEBARGER.

IT has been found by two scientists, working independently, that, by means of thermodynamical considerations, the solubility of a substance may be shown to be the same in all solvents; the mode of deduction employed by each as well as the nature of the experimental proof offered is quite different. Schroeder* established these two equations:

$$l = \frac{\rho}{2}\frac{T_0 - T}{T_0 T}, \qquad (1)$$

and

$$\frac{Q}{T} = \frac{\rho}{T_0} = \text{constant}, \qquad (1\ bis)$$

in which s represents the solubility (defined by the ratio of the number of the molecules of the dissolved substance to the total number of molecules making up the saturated solution); ρ, the latent heat of fusion of a kilogram of the dissolved (solid) substance; T_0, the absolute point of fusion of the dissolved substance; T, the temperature at which saturation takes place; and Q, the heat of solution of a kilogram-molecule of the solid substance in almost saturated solution. Experiments carried out with solutions of *para*-di-brom-benzene in carbon disulphide, benzene, and mono-brom-benzene; of naphthaline in benzene, mono-chlor-benzene, and carbon tetrachloride; of *meta*-di-nitro-benzene in benzene, mono-brom-benzene, and chloroform: corroborated fully the statement that "*the solubilities at equal intervals from the temperatures of fusion for different solid bodies and in different solvents are the same.*"

For alcoholic solutions, however, this law was found not to obtain even approximately; this is undoubtedly due to the circumstance that the alcohols are made up of associated molecules, although Schroeder did not take this view of the matter, perhaps because, at the time of the appearance of his paper, our knowledge of the molecular state of liquids was exceedingly slight.

The law was further tested by Schroeder, by determining the solubility of *para*-di-brom-benzene in mixtures of chloroform and benzene, which was found to be very nearly the same as in either of the pure solvents.

The formula developed by Le Chatelier† runs thus:

* Zeitschr. f. phys. Chem., xi, 449, 1893.
† Comptes rend., cxiii, 638, 1894.

$$0{\cdot}002 \text{ Log nép } s - \frac{L}{t} + \frac{L}{t_0} = 0, \qquad (2)$$

in which s has the same signification as in equation (1); L is the latent heat of solution of a molecule of the dissolved substance in a large amount of the nearly saturated solution; t is the temperature of saturation, and t_0 the point of fusion. This formula leads at once to the conclusion that "*the normal curve of solubility of a given body should be the same in all solvents, because the equation contains no term having reference to the solvent.*"

In a subsequent "Note," Le Chatelier communicated the solubilities of sodium chloride in fused sodium carbonate, and barium chloride; also of lithium sulphate in fused calcium sulphate, lithium carbonate, and sodium sulphate, these data being found entirely in accordance with the law enunciated.

Neither Schroeder nor Le Chatelier claims that the law in question expresses more than an approximate relation between the quantities upon which it has a bearing. Le Chatelier ascribes any exceptions to its generality to differences in the latent heats of solution, which may vary from solvent to solvent. The law should, then, be restricted in its government to chemically similar compounds, and, indeed, it is seen that the compounds, experimented upon by each observer, belong to the same general classes,—the normal organic preparations (Schroeder), and the salts (Le Chatelier). But when we consider groups of chemical substances of differing natures, we perceive that all the regularity of the phenomenon, which has been formulated into a law, disappears; the very fact, that there exist substances, which do not unite with certain others to form the homogeneous mixtures commonly called solutions, is sufficient to warrant the restriction of the law to quite narrow limits. Nearly all inorganic salts are not at all, or, at most, but sparingly soluble in the vast number of organic liquids. Unhappily, our quantitative knowledge of the solubility of salts in organic liquids is very limited; yet perhaps enough data may be collected to permit of the drawing of theoretic conclusions. The object of this paper is to discuss in the light of the Schroeder-Le Chatelier law, the data we possess on the solubility of inorganic salts in normal organic liquids. The importance of this law, which, in its enunciation, is one of the widest-reaching in the domain of solutions, makes it very desirable that it be applied to all cases, in order that it may be ascertained to what degree its approximation towards truth may come.

The greater part of the determinations of the solubilities of salts in organic liquids has been done with the alcohols, espec-

ially ethyl alcohol, as solvents. The choice of such solvents is unfortunate, since, aside from the difficulty experienced in getting and preserving them in a state of purity, they are made up of associated molecules; and this circumstance introduces very serious complications, for not only is the relative proportion of associated molecules different at different temperatures, but also the dissolved substance must have some influence on the degree of molecular association of the solvent, the combined result being that the nature of the solvent varies infinitely. Accordingly, from reasons that are obvious, only "normal" liquids, that is, such liquids as possess the same molecular mass in the liquid as in the gaseous state, will be considered in what follows. By thus eliminating the difficulty arising from the use of associated liquids, it may be possible to get matters into a clearer light. In what follows, no pretensions are made to discuss all the data on the solubility of salts in organic liquids; only such data as seem to have the stamp of reliability will be considered.

Étard* in the course of his extended investigations on the solubility of substances, determined through wide ranges of temperature the solubilities of the salts, mercuric and cupric chlorides, in a number of normal liquids, mostly esters. As in this case the solvents are chemically very similar, it seems likely that, if the above law is at all applicable to the solubilities of salts in organic liquids, Étard's data will permit of its ascertainment.

According to Étard, the solubility of corrosive sublimate in ethyl ether is as follows,—the numbers directly under the temperatures being the number of parts of the salt contained in 100 parts of the saturated solution:

—47°	—40°	—35°	—30°	—19°	0°	13°	83°	100°	115°
5·6	5·8	6·1	5·9	5·6	5.8	5·8	8·4	8·7	9·0

From —47° to +60°, that is, throughout a temperature interval of more than 100°, Étard states that the solubility is the same; the average of the above first seven data is 5·8, which represents the mean solubility for the temperature interval just mentioned. If this be recalculated in molecular proportions, it comes out that 100 molecules of the saturated solution contain 1·65 molecules (of normal size, i. e. corresponding to the formula $HgCl_2$) of the salt. Above 60°, however, the solubility increases with rise of temperature (see Table I for recalculated data).

Similar phenomena were observed in the case of the solubility of corrosive sublimate in acetic ether, the data of which are these:

* Ann. de chim. et de phys., VII, ii, 560.

—50°	—20°	—14°	—6°	0°	7°	19°
39·6	40.5	40·2	40·0	39·5	39·9	40·2

In an interval of more than 70°, the solubility seems to be constant, 100 parts of the saturated solution containing on an average 40·0 parts of salt, or, calculated in molecular proportions, 17·80 molecules of salt are in 100 molecules of the solution. Above 40° the solubility augments with rise of temperature as shown thus:

45°	66°	100°	131°	150°	180°
41·6	44·0	47·8	50·1	57·0	59·3

Étard gives also the solubility of mercury chloride as well as of copper chloride in other ethers; I will not reproduce the data as presented by him, but will throw them into tabular form, after having made the calculations necessary to change them into solubilities defined by the ratio of dissolved molecules to the total number of molecules contained in the solution.

TABLE I.

Solubility of Mercuric Chloride in Organic Liquids.

Names of Liquids,	—50°	—47°	—40°	—35°	—30°	—20°	—19°	—14°	—6°	—3°	—0°	+7°	13°	19°
Ethyl Ether,		1·65	1·65	1·65	1·65		1·65				1·65			
Ethyl formiate,						10·48				10·48				
Ethyl acetate,	17·80					17·80		17·80	17·80		17·80			17·80
Methyl acetate,						16·51								
Amyl acetate,														
Ethyl butyrate,														

TABLE I, Continued.

Names of Liquids,	20°	22°	24°	45°	46°	48°	55°	66°	71°	83°	100°	115°	131°	150°
Ethyl Ether,										2·45	2·52			
Ethyl formiate,			10·48		10·48									
Ethyl acetate,				18·78			20·33				22·92		25·02	30·09
Methyl acetate,			15·56			16·23								
Amyl acetate		9·70				9·82								
Ethyl Butyrate,	5·81						6·26		7·05					

TABLE II.

Solubility of Copper Chloride in Ethyl Acetate and Ethyl Formiate.

	—20°	+20°	24°	37°	40°	50°	72°
Ethyl formiate..	5·88		4·33	4·21		4·08	
Ethyl acetate....		1·98			1·66		0·85

The data given in Tables I and II cannot be said to confirm the predictions of the law under discussion. The solubility of mercuric chloride in acetic ether and in ethyl formiate is indeed nearly the same; and so is that of copper chloride in acetic ether, and of mercuric chloride in ethyl ether. These

cases are, however, exceptional, and can be as likely due to chance as to law. Now, as it may be laid down as a general rule that, no matter in how many instances a "law of Nature" may be confirmed, one exception is sufficient to disprove that generality which is taken to be the very essence of law, it is apparent that the truth of the "law" in the case in hand is very doubtful.

Arctowski* has investigated what may be termed the complement of the case studied by Étard (loc. cit.), for he determined the solubility of three very similar—as regards their chemical constitution—salts, viz: the chloride, bromide, and iodide of mercury, in carbon disulphide. Arctowski communicates his results in the same form as does Étard. To permit of direct comparison with our "law," I have recalculated his results in molecules, the data being given in Table III.

TABLE III.

Solubility of the Halide Salts of Mercury in Carbon Bisulphide.

Mercuric.	$-76\frac{1}{4}°$	$-21°$	$-10\frac{1}{4}°$	$0°$	$8°$	$13\frac{1}{2}°$	$19\frac{1}{4}°$	$25°$	$29°$
Chloride,			0·003	0·004	0·005		0·099	0·011	0·016
Bromide,			0·011	0·018	0·024	0·029	0·038		0·041
Iodide,	0·008	0·013	0·017	0·029	0·039	0·044	0·050	0·056	0·079

Here again we see no confirmation of the "law" in question. The solubilities of the salts are in direct proportion to their molecular masses, and the curves with temperatures on axis of abscissas and solubilities on axis of ordinates are very nearly parallel.

In order to increase our knowledge of the solubilities of salts in organic liquids, and to augment our store of data with which to compare the law in question, I have made a number of determinations of the solubility of several salts in benzene and ethyl ether; the results of this work are given in Tables IV and V.

TABLE IV.

Solubility of Cadmium Iodide, Mercuric Chloride, and Silver Nitrate in Benzene.

Temperatures.	10°·7	11°·4	16°·0	35°·0	38°·8	40°·5
Cadmium iodide,			0·01	0·02		
Mercuric chloride,	0·11	0·12			0·23	0·25
Silver nitrate,				0·01		0·02

TABLE V.

Solubility of Cadmium Iodide in Ethyl Ether.

Temperatures.	0°	15°·5	20°·3
Cadmium iodide,	0·03	0·04	0·05

These data also cannot be said to be favorable to the law.

* Zeitschr. f. anorgan. Chemie, v, 263, 1894.

It is evident from all the data which have been exhibited in the preceding tables, that no trace of the applicability of the Schroeder-Le Chatelier law is to be found. It may, now, be urged that in our definition of solubility as the number of molecules of dissolved substance contained in 100 molecules of the solution, the value of the molecular mass of the dissolved substance has rather arbitrarily been assumed to be equal to that which it has in the gaseous condition, although nothing positive in regard to the real size of the molecule in the dissolved state is known; if the molecular mass of the substance in the gaseous state be doubled, tripled, quadrupled, etc., when it is in solution, the number, expressing the solubility in the manner here adopted, must be changed correspondingly. But even if our knowledge of the molecular state of salts dissolved in organic liquids was sufficient to permit of the introduction of this correction, its amount would not be large enough to account for the very considerable differences in solubility of even the same salt in different, yet chemically similar, organic solvents, as any one can convince himself by a simple calculation. We conclude then, that the law enunciated by Schroeder, and by Le Chatelier, although approximately true for the cases investigated by them, is not applicable to the case of inorganic salts in normal organic solvents.

Chicago, November 26th, 1894.

ART. V.—*Preliminary Notice of the Plymouth Meteorite;* by HENRY A. WARD.

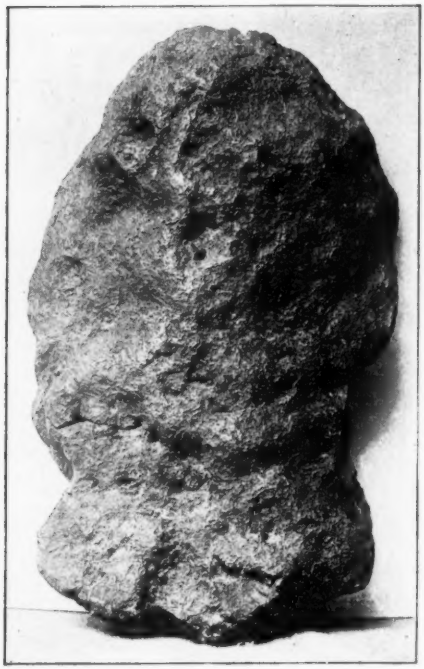
THE Plymouth meteorite was found in the year 1893 by Mr. John Jefferson Kyser, while plowing in a field on his farm about five miles southwest of the town of Plymouth, Marshall County, Indiana. Mr. Kyser had, about the year 1872, found in the same field another, larger mass of the same iron. This mass was pear-shaped, about four feet in length by three feet in its widest diameter, narrowing to six or eight inches at its upper end. It lay for a year or two so near the surface of the ground as to be seriously annoying in plowing the field. On that account, Mr. Kyser, aided by his son, dug a deep hole by the side of the mass and buried it to the depth of one and one-half to two feet beneath the surface, where it should thenceforth do no more damage.

The account of this I had last June from the son, Mr. John M. Kyser, now city clerk of Plymouth. Mr. Kyser well remembers the circumstance of the finding of the large piece and assisting his father in burying the same; and he further

thought that, notwithstanding the removal of certain landmarks (a fence and tree), in the field, he would still be able to locate it very closely. This he subsequently undertook to do by trenching, but was unsuccessful in finding the mass. I was, myself, present and assisted in a further search for it in September last, using a surveyor's magnetic needle, with the hopes of the same being attracted to the mass and discovering it, but all to no purpose. Mr. Kyser seems to feel very confident of his knowledge of the immediate vicinity of the mass where he buried it 22 years ago, but is unable to prove its presence by re-discovery. Nor has he the aid of another eye-witness, his father having died soon after the original finding and burying as above mentioned.

The smaller piece which was, as before said, found in 1883, was presented by Mr. Kyser, Sr., to Mr. W. S. Adams who, at that time kept a plow factory in the city of Plymouth. It was retained in their family until last November, when it was brought to Ward's Natural Science Establishment in Rochester, N. Y. by Mrs. Adams from whom I procured it.

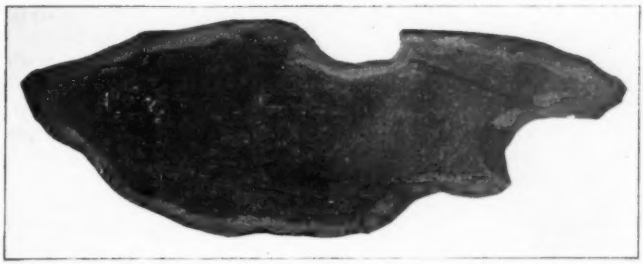
1.



The mass, as represented in the accompanying cut (fig. 1) is a lengthened, tongue-like form, not unlike a rude Mound-builder's axe. Its greatest length is $12\frac{1}{2}$ inches; its width $7\frac{3}{8}$ inches; its thickness in the middle about two inches, from which, in the greater part of its length it slopes in a somewhat even manner to a thin, rounded edge.

Its surface is deeply eroded by oxidization, so that, although sound and free from scales, it shows no signs of an original crust. The characteristic pittings of meteorites are also by the same cause rendered somewhat feeble, although still quite clearly visible. We have cut a number of thin slices from the mass. These etched in dilute nitric acid give very clear Widmanstätten figures, which are well shown in the accompanying cut (fig. 2.) There are, further, several small nodules of troilite.

2.



A careful analysis of this iron has been very kindly made for me by Mr. J. M. Davison of the Reynolds Laboratory of the University of Rochester, and I give the same below.

Analysis of Plymouth Meteorite.

Fe	88·67
Ni	8·55
Co	0·66
Cu	0·24
P	1·25
Graphite	0·11
S	0·07
	99·55

This iron, herein briefly noticed, is interesting in many ways, and it is much to be regretted that the large mass, of which the record seems to me to be entirely reliable, cannot be re-discovered.

SCIENTIFIC INTELLIGENCE.

I. Chemistry and Physics.

1. *On Diammonium.*—The suggestion of Curtius, the discoverer of hydrazine or diamide, that there should be a hypothetical radical diammonium which bears the same relation to diamide that ammonium does to ammonia, has been verified by this chemist in connection with Schrader; they having prepared a large number of double salts containing this diammonium radical. Diamide itself H_2N-NH_2 is extremely unstable and its separate existence is yet somewhat uncertain; while its hydrate $N_2H_4 . H_2O$ is very permanent. Moreover, diamide is also unlike ammonia in the fact that it is a diacid base while ammonia is monacid. The normal hydrazine chloride is ClH_3N-NH_3Cl and the sulphate is $(H_3N-NH_3)''SO_4$. The analogy thus shown between diammonium and the bivalent metals of the alkali-earths, is further strengthened by the sparing solubility of its sulphate and its inability to form alums with the sulphates of the alumina group. On the other hand, however, certain properties of diammonium show that it resembles closely the alkali metals. Thus its hydrate generally acts as a monacid base. Its chloride $N_2H_6 . Cl_2$ is decomposed below 100° into hydrogen chloride and $N_2H_4 . HCl$, which cannot be made to lose more hydrogen chloride without destruction of the base. The hydrate $N_2H_4 . (H_2O)_2$ can exist only in solution. On evaporation it passes into the hydrate $N_2H_4 . H_2O$, which boils without decomposing. Ammonia gas displaces only half the acid of the sulphate $N_2H_4 . H_2SO_4$ leaving the stable sulphate $(N_2H_4)_2H_2SO_4$. Moreover only one nitrate $N_2H_4 . HNO_3$ appears to exist and only one thiocyanate $N_2H_4 . SCNH$. The authors conclude that diammonium may act as a univalent radical $(N_2H_5)''$ and also as a bivalent one $(N_2H_6)''$, the salts of the former being the more stable. Double salts of diammonium sulphate with the sulphates of copper, nickel, cobalt iron, manganese, cadmium and zinc have been obtained, all of which are anhydrous. They are readily thrown down as precipitates on mixing strong solutions of the metallic sulphate and diammonium sulphate. The latter may be either the sulphate $N_2H_6SO_4$ or $(N_2H_5)_2SO_4$. While the former of these sulphates is difficultly soluble the latter is deliquescent; yet the sparingly soluble double salts always contain the latter sulphate. Moreover salts of the type $R''SO_4 . (N_2H_4)_2$ and $R''SO_4 . (N_2H_4)_3$ have been obtained, the former containing zinc or cadmium, the latter nickel or cobalt. All attempts to obtain alums containing the diammonium sulphate $N_2H_6 . SO_4$ have been unsuccessful.—*J. prakt. Ch.*, II, l, 311, September, 1894. G. F. B.

2. *On Nitrogen Trioxide.*—The actual existence of the trioxide of nitrogen or nitrous anhydride N_2O_3, appears to have been established by Lunge and Porschnew. Although the oxides

N_2O_2 and N_2O_4 show scarcely any tendency to unite at the ordinary temperature, the authors find that at $-21°$, these oxides combine in exact molecular proportions to form N_2O_3 which condenses as an indigo-blue liquid; 98·3 per cent of the trioxide having been obtained in one experiment. It is perfectly stable at and below this temperature; but at a slightly higher temperature, even under pressure, it begins to decompose, the dissociation becoming almost complete on conversion of the liquid into gas. Hence it would seem that the trioxide is not capable of existing in the gaseous state; although certain facts observed in the investigation seem to indicate that a residue of N_2O_3 molecules may escape dissociation and may exist side by side with the molecules of N_2O_2 and N_2O_4 into which the trioxide is decomposed.—*Zeitschr. anorg. Chem.*, vii, 209, September, 1894.

G. F. B.

3. *On the Physical Properties of Nitrogen Monoxide.*—The physical properties of carefully purified nitrogen monoxide have been studied by Villard. The gas was obtained pure either by the decomposition of its hydrate or by fractioning the liquid oxide; the gas in the latter case being passed through suitable purifying and drying agents. It was then liquefied and allowed to boil to expel the dissolved gases. As thus obtained, it is free from the less liquefiable gases, its maximum vapor pressure is independent of the volume of the vapor, and a small increase of pressure causes complete liquefaction. The densities of the gas and of the liquid at various temperatures are given as follows:

Temperature	0°	5°	10°	17·5°	26·5°	32·9°	34·9°	36·3°
Density of liquid..	0·9105	0·885	0·856	0·804	0·720	0·640	0·605	0·572
Density of gas....	0·0870	0·099	0·114	0·146	0·207	0·275	0·305	0·338

The critical temperature of pure N_2O is 38·8°, the critical volume is 0·00436, the critical density 0·454 and the critical pressure 77·5 atmospheres.—*C. R.*, cxviii, 1096, May, 1894. G. F. B.

4. *On the use of the Refractive Index for determining Critical Temperatures.*—By a careful observation of the interference bands, Chappuis has been able to note the changes which take place in the index of refraction of a liquefied gas in the vicinity of its critical temperature. For this purpose the liquid was contained in a cylindrical cavity in a steel prism, having apertures closed by optically plane and parallel glass plates; the whole being immersed in a liquid whose temperature could be maintained constant. By means of a pair of Jamin mirrors, two beams of light, starting from a Billet compensator, pass through the liquid in the prism, traversing in their course the enveloping bath, the sides of which also are made of plane parallel glass. When liquid carbon dioxide is used, the refractive index is constant and the bands remain stationary; beyond this temperature the refractive index increases rapidly and the bands fall. At 31·61° the curve of the index shows a vertical tangent; and the intersection of this curve with the straight line which represents the index above this temperature, is the critical point of the

index. The uncorrected results vary between 31·60° and 31·62° only, and the corrected value for the critical point is given as 31·40°; in close agreement with Amagat's value 31·35°.—*C. R.*, cxviii, 976, April, 1894. G. F. B.

5. *On the Constants of Refraction of Carbonyl Compounds.*—In order to ascertain the influence exerted by the presence of a number of carbonyl groups in a molecule upon its molecular refraction, NASINI and ANDERLINI have determined the constants of refraction of several carbonyl compounds. Mond and Nasini had attributed the exceptionally high molecular refraction of nickel tetracarbonyl to the fact that in this substance nickel is an octad. But Gladstone had expressed the opinion that the equally high molecular refraction of ferropentacarbonyl, is due rather to the peculiar arrangement of the carbonyl groups than to the presence of iron as a decad. The substances examined were quinone, diacetyl, dipropionyl, tetrachlorotetraketohexamethylene, dibromodichlorotetraketohexamethylene, leuconic acid and potassium croconate; the last two being examined in solution in water, and the others (excepting the second and third) in solution in benzene. The measurements were made for the line $H\alpha$ and the results calculated both for the formula $M(\mu-1)/\delta$ and the formula $M(\mu^2-1)/(\mu^2-2)\delta$. It was observed that the experimental values agreed well with the calculated ones. If, however, the atomic refraction of potassium be taken as 8·1 as given by Gladstone, the observed molecular refraction of potassium croconate becomes 58·20 for the first formula given above; so that if the ordinary constant be assigned to the carbonyl groups, the atomic refraction of potassium must be taken as 22·5 for the first, and 12·40 for the second, of these formulas. Since the measurements made on the other carbonyl compounds show that the presence of several carbonyl groups causes no abnormal increase in the molecular refraction, the authors attribute the abnormally high molecular refraction of this substance to the metal alone; though they admit that the anomalous results sometimes obtained with such compounds may be due in part to the fact that they were made in solution.—*Gazzetta Chim. Ital.*, xxiv, i, 157; *J. Chem. Soc.*, lxvi, ii, 301, August, 1894. G. F. B.

6. *On the Electrolysis of Copper Sulphate in Vacuo.*—It is a well recognized fact that the electrolytic deposition of copper from a solution of its sulphate does not conform rigorously to Faraday's law; Gray having shown that the deposit of metal is heavier the higher the current density and the lower the temperature. This appears to be due to the fact that copper is slightly soluble in a copper sulphate solution. Since Schuster had suggested that this solution of the copper is due to the oxygen present in the copper sulphate solution, GANNON has made a comparison between the masses of copper deposited in two voltameters in series, from one of which the air was exhausted. His results show that with a neutral solution, the deposit of copper in the vacuum tube is higher than that in the one under the atmos-

pheric pressure; although the percentage difference is not constant. On adding a little free sulphuric acid to the air solution, however, this difference becomes more constant and is higher than before. On adding the acid to both voltameters the percentage difference becomes constant within the limits of experimental error. In this case, where the current density rises above 0·01 ampere per square centimeter of active kathode, there is no practical difference between the two deposits. For densities below this value however the vacuum deposit is appreciably higher than the air deposit. If a curve be drawn to represent the deposits obtained in a vacuum at different current densities, it will be observed to be more regular than the air curve, and to be approximately a straight line for densities below 0·01 ampere per square centimeter.

In a paper following this, SCHUSTER gives the details of certain experiments made by him some years ago, proving that when copper is placed in a copper sulphate solution containing free sulphuric acid, and the tubes are exhausted of air, the diminution in the weight of the copper is quite insignificant compared with what takes place in the presence of air. In sulphuric acid alone the metal behaves similarly.—*Proc. Roy. Soc.*, lv, 66, 84, January, 1894. G. F. B.

7. *On the Propagation of Electromagnetic Waves in Ice and on the Dielectric Power of this Substance;* by M. BLONDLOT.—In a previous note (*Comptes Rendus*, July 25, 1892) I enunciated the following proposition:—The length of the waves which an electromagnetic oscillation can emit is the same whatever be the insulating medium in which the experiment is made; in other words, the wave-length depends on the oscillator alone, just as in acoustics the wave-length of a pipe depends only on the length of the pipe.

The confirmatory experiments described in the Note cited referred to oil of turpentine and to castor oil; the law holds perfectly for both these substances, and everything leads to the belief that this will be the same for other dielectrics.

There is, however, a doubt about ice, in consequence of the exceptional properties ascribed to it. The experiments of M. Bouty (*Comptes Rendus*, March 7, 1892) show in fact that ice has a dielectric power of 27, that is to say incomparably greater than that of all other substances. Suspecting that the law relative to the propagation of waves might not apply to a dielectric so different from the others, I resolved to submit the question to experiment.

For these investigations I availed myself of the intense and prolonged frosts of the winter of 1892–93. M. M. Dufour has helped me in carrying them out, which the rigour of the cold rendered difficult and even painful. I thank him for his extreme kindness on this occasion.

The method which I adopted was the following, which, with slight modifications necessitated by the solid character of the

dielectric, is the same as that I used in the case of turpentine and castor oil.

Electromagnetic waves were transmitted along two tinned copper wires 2·5 millim. in diameter stretched horizontally and parallel to each other at a distance of 0·8 meter. A resonator of gilt copper is placed in a fixed position between the wires; the portion of the transmitting wires beyond the resonator is contained in a wooden trough 4 meters in length. The trough being filled with liquid, the position is sought at which a movable bridge must be placed, joining the wires beyond the resonator to cause the spark to disappear; the distance from the bridge to the resonator is then a quarter of the specific wave-length of the resonator; the position of the bridge is accurately noted.

That done, I surround the part of the resonator forming the condenser with a water-tight bag of parchment-paper which I fill with distilled water, and then freeze this water; the layer of air is thus replaced by one of ice. Measuring the wave-length afresh, it is found to be considerably greater than in the first experiment, having become $\frac{141}{100}$ of what it was.

The trough is then filled with water which is frozen, and then the position of the bridge for disappearance of the spark is again sought. For this purpose the ice at the distant end of the trough is broken and progressively removed. I ascertained that this position is *exactly* the same as in the first case, when the dielectric was air.

The experiment four times repeated, varying each time the capacity of the condenser, always gave the same result. The proposition relative to the wave-length is therefore true for ice as well as for other dielectrics. Hence, as shown in my previous Note, Maxwell's relation that the dielectric power is also equal to the square of the refractive index also holds for electromagnetic waves in the case given.

The preceding results, partly unforeseen, led me to determine the dielectric constant of ice, using electromagnetic undulations. The experiment cited above gave all the data necessary for this determination.

For if λ and λ' are the wave-lengths corresponding to a given resonator, working respectively in air and in a substance of dielectric power K, we have

$$\frac{\lambda'}{\lambda} = \sqrt{K}.$$

As stated above, I found

$$\frac{\lambda'}{\lambda} = \frac{141}{100},$$

whence

$$K = 2 \text{ in round numbers.}$$

The experiment repeated a dozen times always gave the same result. I consider the relative error does not exceed $\frac{1}{20}$, for the

plate of ice was almost entirely free from air bubbles. According to this, ice does not present exceptional dielectric properties.

It remains to be explained how MM. Bouty and A. Perot obtained values of a totally different order for the dielectric power of ice. In the first place, in M. Bouty's method the charge and discharge were enormously slower than in my experiments. Is it not probable, then, that the physical magnitudes measured by M. Bouty and myself were in themselves very different. In any case we know at present too little about the dielectric properties of bodies to be surprised at the divergences of numbers obtained by two methods so dissimilar, however great they are.—*Comptes Rendus*, October 8, 1894.—*Phil. Mag.*, xxxviii, 578, Dec., 1894.

8. *Rotation of magnetic lines.*—E. LECHER describes in the Wiener Berichte an experiment performed to test whether, when a magnet turns about its magnetic axis, the lines of force remain at rest or turn with the magnet. A magnet was divided by an equatorial plane into two parts which could turn independently. It was possible to obtain from the two extremities of the magnet an induced current of such a magnitude as cannot be explained by the cutting of the rotating lines of force by the extremely short brushes employed. These currents can be explained if we suppose that a rotating magnet cuts its own lines of force which remain fixed in space.—*Nature*, Nov. 22, 1894, p. 84.

J. T.

9. *Magnetization of iron and nickel wire.*—KLEMENCIC has measured the permeability of iron steel and nickel wire with the rapidly alternating currents employed by Hertz. He employed the method of a thermal junction previously described by him and obtains the following values.

Soft iron	$\mu = 118$
Hard steel	$\mu = 115$
Bessemer steel	$\mu = 77$
Nickel	$\mu = 27$

—*Ann. der Physik und Chemie*, No. 12, pp. 705–720. J. T.

10. *New Storage Battery.*—In researches on the condensation of electrolytic gases by porous bodies, particularly by metals of the platinum group, M. L. CAILLELET and E. COLLARDEAU find that platinum and palladium in the spongy condition and ruthenium, iridium and gold in the finely divided state form poles which condense electrolytic gases and hence produce a gas battery, which on connection of the poles is capable of giving up the stored energy for a short period. The storage capacity is vastly increased by subjecting the poles to great pressure during charging. With spongy platinum and iridium a storage capacity can be obtained which is greater than the capacity of lead accumulators per unit of weight. With finely divided palladium, a storage capacity of 176 ampere-hours per kilograms of palladium was obtained at a pressure of 600 atmospheres. The storage capacity of an ordinary lead accumulator is about 15 ampere-hours per kilogram of lead.—*Comptes Rendus*, Nov. 12, 1894.

J. T.

11. *Resemblances between the grouping of figures on soap films and the arrangement of stars and nebulæ.*—In a leading article, Professor QUINCKE, of Heidelberg, draws attention to the remarkable resemblances between the figures produced on oily films by the operation of water, and the grouping of stars and nebulæ. He suggests that the tendency of modern physics is to ignore any qualitative differences between infinitely great and infinitely small distances. The great masses of the fixed stars in infinite space and the little masses of the infinitely near molecules in the soap films must react upon each other according to the same laws. The protoplasm of organic nature also resembles in structure and movement phenomena, the structure and movements observed on oily films.—*Ann. der Physik und Chemie*, No. 12, 1894, pp. 593–632. J. T.

II. GEOLOGY AND MINERALOGY.

1. *Glacial succession in Europe.*—The following quotations from the third edition of Geikie's The Great Ice Age,* will present to our readers in concise form the latest views of the author upon the glacial succession in Great Britain and Europe, and the chief data upon which the interpretation is based.

I. PREGLACIAL TIMES.†—The earliest indications of the approaching Ice Age are met with in the marine deposits of the Pliocene system. The older Pliocene deposits introduce us to a time when the waters of the North Sea were tenanted by a fauna which is clearly indicative of genial climatic conditions. And similar testimony to the warmth of the period is furnished by the contemporaneous marine lacustrine and terrestrial accumulations of other regions of Europe. In those days the sea occupied considerable tracts in the east and south of England, in Belgium, Holland, Northern and Western France, and the coast-lands of the Mediterranean. As time rolled on, however, the genial conditions gradually passed away. The southern forms slowly retreated from the North Sea, while at the same time northern and boreal types came to occupy their place. Similar migrations were in progress farther south, many British and boreal forms finding their way into the Mediterranean. Upon the land like changes

* The Great Ice Age and its relation to the Antiquity of man; by James Geikie, third edition, largly rewritten, pp. 850. (Edward Stanford). London, 1894.

† The general reader will quite understand what is meant by the term "preglacial," but I ought to mention that it has been applied by geologists to certain deposits which underlie the "lower boulder-clay" of the regions in which these deposits occur. But, as we have learned, many accumulations of so-called "lower boulder-clay" are not the products of the earliest epoch of glaciation, and the fossiliferous beds which sometimes underlie them are, therefore, not necessarily of preglacial age. I apply the term exclusively to deposits which were laid down before the earliest appearance of glacial conditions in temperate latitudes. These, so far as our present knowledge goes, are the only accumulations which we are justified in classing as preglacial.

were brought about—the luxuriant flora and the great mammals of the Pliocene retreating gradually before the approaching winter of the Glacial Period.

II. First Glacial Epoch.—Eventually a thoroughly arctic fauna lived in the North Sea. Great snow-fields at the same time came into existence, and a gigantic glacier occupied the basin of the Baltic.* The mountainous parts of the British Islands, we can hardly doubt, must likewise have been ice-clad, but of this there is no direct evidence. Farther south the Alpine Lands were swathed in snow and ice, and great glaciers occupied all the mountain-valleys and piled up their terminal moraines upon the low-grounds at the foot of the chain. In Central France very considerable glaciers also descended from the great volcanic cones of Auvergne and Cantal, and deployed upon the plateaux. And probably in many other mountain-districts similar conditions obtained.

III. First Interglacial Epoch.—Eventually cold conditions passed away. The arctic fauna retreated from the North Sea, and at the same time dry land occupied the southern part of that sea up to the latitude of Norfolk at least. Across this new-born land flowed the Rhine and other rivers. A temperate flora, comparable to that now existing in England, clothed the land in our latitude, while the hippopotamus, elephants, deer, and other mammals became denizens of our country. In other parts of Europe similar genial conditions obtained—conditions which, to judge from the flora, were even more genial than are now experienced in the same regions. A luxuriant deciduous flora occupied the valleys of the Alps, and flourished at heights which it no longer attains. That flora was accompanied by a mammalian fauna (North Italy) which embraced among other forms *Elephas meridionalis*. From the amount of river-erosion effected during this epoch we may gather that the stage was one of long duration. By-and-by, however, cold conditions again supervened—the temperate flora disappeared from England, and was gradually replaced by arctic forms.

IV. Second Glacial Epoch.—The appearance of that arctic flora and the immigration into the North Sea of arctic mollusks heralded the approach of the greatest of the European ice-sheets. This enormous *mer de glace* covered all the northern part of the Continent and flowed south into Saxony. At the same time the Alpine glaciers reached their greatest extension, while in all the other mountains of Europe snow-fields and glaciers made their appearance. In extraglacial tracts, as in Southern England and Northern France, and in many other regions, the formation of rock-rubble was in active progress, and much movement of such superficial accumulations took place. These physical changes

* The limits reached by this earliest "great Baltic glacier" are not known. In Southern Sweden, however, it occupied a wider area than the great Baltic glacier of the fourth glacial epoch, its northern limits lying at least thirty miles farther north than those reached by the latter.

were necessarily accompanied by great migrations of flora and fauna—arctic alpine plants coming to occupy the low-grounds of Central Europe, and northern animals ranging down to the shores of the Mediterranean.

V. Second Interglacial Epoch.—The enormous morainic accumulations and fluvio-glacial gravels of the second glacial epoch are sufficient evidence of its prolonged duration. Eventually, however, it passed away and the climate by degrees became temperate and even genial. The arctic-alpine flora and northern fauna retreated from the low-grounds and were replaced by temperate and southern forms. The character of the plants which then occupied North Germany and Central Russia is suggestive of a milder and less extreme climate than is now experienced in those regions—and the mammalian fauna of the epoch, which included the hippopotamus and *Elephas antiquus*, was in keeping with the flora. Britain would appear to have been connected with the Continent at this time, and land-passages probably joined our Continent to North Africa. Again, however, the climate began to deteriorate, accompanied by renewed migrations of flora and fauna, and as the third glacial epoch approached, much low-lying land in Northern and Northwestern Europe was submerged. The long duration of this interglacial phase is shown by the great depths to which the rivers of the epoch succeeded in eroding their valleys.

VI. Third Glacial Epoch.—At the climax of this epoch a most extensive ice-sheet again overwhelmed the major portion of the British Isles and a vast area of the Continent. It did not, however, attain the dimensions of its predecessor. From the Alps great glaciers again descended to the low-grounds, where they dropped the terminal moraines of the "inner zone." These moraines form most conspicuous objects, and extend in curving lines between the highly denuded moraines of the first and second glacial epochs. Like these they are accompanied by well-marked sheets and terraces of fluvio-glacial gravels. Many of the other mountains of Europe were similarly snow-clad and glaciated, while rubble-drifts accumulated in extraglacial regions.

VII. Third Interglacial Epoch.—The third glacial epoch eventually passed away like its predecessors and was gradually succeeded by temperate conditions. Of this change the most direct evidence we have is that furnished by the youngest interglacial beds of the Baltic coast-lands. It is probable, however, that a considerable proportion of the old alluvial deposits of Britain and Ireland, which have hitherto been classed as of post-glacial age, really belong to this interglacial epoch. Amongst these are the clays, etc., with Irish deer, red-deer, etc., which underlie the older peat-bogs. In the Baltic area the interglacial beds contain in some places arctic forms, in others a temperate marine fauna, in yet others they have yielded mammoth, woolly rhinoceros, horse, Irish deer, and urus. It is obvious that these beds cannot be strictly contemporaneous. Some probably belong to

the beginning, others to the middle of the third interglacial epoch, while others again may pertain to its close. They show us clearly, however, that after the disappearance of glacial conditions the Baltic became tenanted by a temperate North Sea fauna, while the adjacent lands supported a corresponding terrestrial fauna and flora.

VIII. Fourth Glacial Epoch.—In the early stages of this epoch the low-grounds of Scotland were submerged to the extent of 100 ft. at least, while an arctic marine fauna lived round the coasts. Eventually the various mountain-districts of our islands were cased in snow and ice, large glaciers filling all the Highland fiords and calving their icebergs in the sea—a condition of things implying a snow-line not exceeding 1,000 to 1,600 ft. in elevation. But the greatest development of ice was witnessed in the Baltic area. The Scandinavian peninsula once more supported an ice-sheet that broke away in icebergs at the mouths of all the fiords of Western Norway. Finland was well-nigh overwhelmed, while the Baltic basin was occupied by a great ice-stream which invaded North Germany and Denmark. Later on, as the ice-sheet melted away, a wide area in Scandinavia was submerged in a cold sea which communicated widely with the Baltic. In the Alps large glaciers flowed for long distances down the great valleys, but came far short of attaining the dimensions reached by those of the preceding glacial epoch. At the same time small local glaciers appeared in the high valleys of some of the mountain-ranges of Middle Europe.

IX. Fourth Interglacial Epoch.—At the climax of this epoch temperate conditions prevailed, and extensive forests of deciduous trees spread far north into regions where such trees no longer flourish. The British Isles now formed part of the Continent. The cold sea had retreated from Scandinavia, and the Baltic was converted into a great lake. Eventually, however, submergence again ensued, but the sea, which now invaded Scandinavia and communicated with the Baltic, was tenanted by a fauna indicative of more genial conditions than obtain at present.

X. Fifth Glacial Epoch.—This epoch is characteristically represented by local or valley moraines in the British Isles, the position of which indicates that the snow-line reached in Scotland an average height of 2,500 ft. The disappearance of the genial conditions of the preceding temperate epoch was marked by the submergence of the Scottish coast-lands to a depth of about fifty feet below their present level. Here and there in the northwest of Scotland, glaciers reached the sea and dropped their moraines on the beaches of the period. To the same epoch may be assigned the formation of most of the corrie rock-basins of the British Isles—each of these basins marking the site of a glacier of more or less limited size. The submergence in Scotland probably commenced in the preceding temperate epoch, as it did likewise in

Scandinavia. In the Alps the fifth glacial epoch is marked by the moraines of the so-called "second postglacial stage"—moraines that indicate a renewed advance of the glaciers of that region.

XI. Fifth Interglacial Epoch.—This epoch was heralded by the re-emergence of the land and the retreat of our valley-glaciers. Again the British area attained a wider extent than at present, but we cannot tell whether it became united to the Continent. The upper "buried forests" in the peat-bogs of Northwest Europe show that this epoch was characterized by drier conditions and a remarkable recrudescence of forest-growth—conditions recalling those of the preceding interglacial epoch.

XII. Sixth Glacial Epoch.—To this epoch belong our latest raised-beaches, which indicate a limited submergence not exceeding, in Scotland, twenty to thirty feet. The climate once more became humid and less favorable to forest-growth. Hence forests decayed while peat-bogs extended their area. The snow-line stood at an elevation in Scotland of 3,500 ft.—and thus nourished a few small glaciers in our loftiest mountain-groups.

XIII. The Present.—Marked in Britain by the retreat of the sea to its present level, and by the return of milder and drier conditions and the final disappearance of permanent snow-fields.

It will be observed that in this short résumé I have abstained from the use of the term "postglacial." The word has been applied to deposits of such widely different ages that it has ceased to be of any value for classificatory purposes. From late Pliocene down to the close of the Pleistocene time we have the record of a continuous series of geographical and climatic changes. Early in the cycle the glacial and interglacial phases attained their extreme development. The climax once passed, each successive cold and genial epoch declined in importance. In a word, the climatic and geographical changes became less and less marked as the cycle drew to a close. From the point of view of the present it seems absurd to speak of "fifth" and "sixth" glacial epochs, seeing that these epochs were marked only by the presence in Britain of limited snow-fields and small local glaciers. When either of these epochs is contrasted with the conditions that obtained in early glacial times they would seem to be more properly described as temperate epochs. But they undoubtedly belong to one and the same series of alternating cold and genial conditions, and from this larger point of view cannot be designated otherwise than glacial.

In the following tabular statements the glacial and interglacial succession is given in ascending order, beginning with the lowest beds—the successive stages being indicated in the several tables by the same numbers.

GLACIAL SUCCESSION IN BRITISH ISLES.

1. Weybourn crag and Chillesford clay.	1. Marine deposits with pronounced arctic fauna.
2. Forest-bed of Cromer.	2. Temperate flora; *Elephas meridionalis, E. antiquus, Rhinoceros etruscus, Hippopotamus, &c.*
3. Lower boulder-clays and associated fluvio-glacial deposits.	3. Ground-moraines, &c., of most extensive ice-sheet.
4. Marine, fresh-water, and terrestrial accumulation; basin of Morav Firth; basin of Irish Sea; Lanarkshire, Ayrshire, Edinburghshire, &c.; Hessle gravels; Sussex beach-deposits, &c.; Settle Cave, &c.	4. Northern and temperate flora and fauna; *Elephas primigenius, Rhinoceros tichorhinus;* reindeer, &c.; hippopotamus, *Elephas antiquus, Rhinoceros leptorhinus,* Irish deer, grisly bear, lion, hyæna, &c.
5. Upper boulder-clay and associated fluvio-glacial deposits.	5. Ground-moraine of ice-sheet which extended south to the Midlands of England.
6. Fresh-water alluvia underlying oldest peat-bogs: probably a considerable proportion of our so-called "postglacial" alluvia.	6. Temperate flora and fauna; Irish deer, red deer, *Bos primigenius,* &c.
7. Boulder-clays and terminal-moraines of mountain-regions; 100-ft. beach of Scotland; arctic plant-beds.	7. Morainic accumulations of district- and large valley-glaciers; arctic marine fauna; snow-line at 1,000 to 1,600 ft.; arctic flora.
8. Lower buried forest.	8. Temperate flora and fauna.
9. Peat overlying "lower buried forest"; Carse-clays and raised beaches; valley-moraines and corrie-moraines.	9. Small glaciers in mountain-regions; snow-line at 2,400 to 2,500 ft.
10. Upper buried forest.	10. Temperate flora and fauna.
11. Peat overlying "upper buried forest"; low-level raised beaches; high-level valley-moraines and corrie-moraines.	11. Small glaciers in the most elevated regions; snow-line at 3,500 ft.

GLACIAL SUCCESSION IN NORTHERN EUROPE.

1. Lowest boulder-clay of Baltic coast-lands.	1. Ground-moraine of earliest Baltic glacier.
2. Alluvia underlying lower diluvium of Hanover, Brandenburg, &c.	2. Temperate flora and fauna, including *Cervus elaphus, C. capreolus, Bos,* sp., *Rhinoceros,* sp., &c.
3. Lower diluvium of Central and Southern Germany, Holland, Central Russia.	3. Ground-moraines, &c., of most extensive *mer de glace.*
4. Fresh-water and marine deposits of Baltic coast-lands, Grünenthal, Rixdorf, Kottbus, Moscow, &c.	4. Northern and temperate floras and faunas; mammoth, *Elephas antiquus,* Irish deer, horse, &c.
5. Upper diluvium of Central Germany, Poland, West Central Russia; second boulder-clay of Baltic coast-lands.	5. Ground-moraines, &c., of ice-sheet which flowed some 40 or 50 miles south of Berlin.
6. Fresh-water and marine deposits of Baltic coast-lands.	6. Northern and temperate floras and faunas.
7. Youngest boulder-clay of Baltic coast-lands; terminal moraines in South Norway and Sweden, Baltic Ridge, and Finland. Arctic plants under oldest peat-bogs of Denmark, Norway, Sweden, &c.; *Yoldia*-clays, &c.	7. Accumulations of last great Baltic glacier; submergence in Scandinavia; adjacent lands clothed with arctic flora.

8. *Ancylus*-beds of Baltic area; older buried forests generally; *Littorina*-beds of Baltic area in part.	8. Temperate flora and fauna; climate at climax of stage more genial than now; conditions become more humid in later stages.
9. Peat overlying older buried forests; calcareous tufas, &c.; *Littorina*-beds in part; large valley-moraines in Norway.	9. Humid climate; glaciers here and there reach heads of fiords.
10. Younger buried forests.	10. Temperate flora and fauna; drier conditions.
11. Peat overlying younger buried forests; high-level terminal moraines in Norway.	11. Humid climate; relatively small glaciers.

GLACIAL SUCCESSION IN CENTRAL AND SOUTHERN EUROPE.

1. Lowest ground-moraines; terminal moraines opposite mouths of great valleys; plateau-gravels (Alpine Lands). Oldest morainic accumulations of Central France (?).	1. Accumulations of first glacial epoch.
2. Hötting breccia; lignites of Leffe, &c.; so-called Upper Pliocene alluvia of Central France.	2. Flora indicates warmer conditions than now obtain in those regions; *Elephas meridionalis, Rhinoceros leptorhinus*, &c.; valleys deeply eroded.
3. Ground-moraines; terminal-moraines of outer zone; high-level gravel terraces (Alpine Lands); "Ceppo" of North Italy; Pleistocene conglomerate of Ligurian coast-lands; torrential gravel terraces of South-west Italy; erratics of raised beaches (France); older moraines of Urals, Carpathians, and mountains of Central Europe; of Central France, Pyrenees, Corsica, Appenines, Balkan Peninsula, &c.; lower breccias of Gibraltar; and rubble-drift, in part, of other regions.	3. Accumulations of epoch of maximum glaciation.
4. Lignites of Switzerland, Bavaria, &c.; alluvial deposits; beach accumulations underlying rubble-drift (Northern France); terraces cut in older breccias of Gibraltar.	4. Temperate flora; *Elephas primigenius, E. antiquus, Rhinoceros Merckii, Bos primigenius, Cervus elaphus*, &c.; valleys deeply eroded.
5. Ground-moraines; terminal moraines of inner zone; low-level gravel terraces (Alpine Lands); valley-moraines in other mountain-regions; rubble-drifts in part.	5. Accumulations of third glacial epoch.
6. Alluvial deposits, &c. (?)	6. Not yet recognized.
7. Terminal moraines in large longitudinal valleys of the Alps; small valley-moraines in the higher valleys of other mountain-regions.	7. Accumulations of fourth glacial epoch = "first postglacial stage" of glaciation.
8. Alluvial deposits, &c. (?)	8. Not yet recognized.
9. Terminal moraines in higher valleys of Central Alps.	9. Accumulations of fifth glacial epoch = "second postglacial stage" of glaciation.
10. Alluvial deposits, &c.	10. Not recognized.
11. High-level moraines in Western Alps.	11.

2. *Changes of level in the region of the Great Lakes in recent geological time;* by FRANK B. TAYLOR. (Letter to J. D. Dana.)—"*First*, as to the probable place of the highest shore line on the north coast of Lake Superior. It seems to me that when we take full account of all that is now known about the highest shore line in the regions adjacent to Lake Superior we are bound to conclude, pending further exploration, that this line probably attains an altitude of at least 1200 to 1300 feet above sea level in the Nipigon region at the north-northwestern extremity of the lake, and at least 1100 to 1200 feet on the northeastern side at Lake Missinaibi. The facts upon which this opinion rests are as follows: The altitude of the Herman beach of "Lake Agassiz" (ancient Gulf of Winnipeg) is 1055 feet at Lake Traverse, and 1220 feet at the international boundary. On the east side of Red river in Minnesota this beach rises a little more rapidly. On the southeast side of Beltrami island in northeastern Minnesota it occurs at 1087 feet and on the northwest side at 1195 feet. At Duluth, 1134 feet, and at Mt. Josephine on the northwest coast of Lake Superior, 1207 feet. Near L'Anse and again near Marquette on the south coast, 1190 feet. Near Sault Ste. Marie 1014 feet. Near Cartier, Ontario, about 1200 feet, and at the ancient Nipissing strait at North Bay, Ontario, 1140 feet on the north and more than 1220 feet on the south side.

It is noticeable that the lowest level of the line in this list is at Sault Ste. Marie. If the highest shore line passes downward to the north from this place, or even on a level, it must cut off the supposed straits to the north of Lake Superior. But considering the high altitude of the shore line in all the regions bordering the lake, Green Bay only excepted, the chance of this seems extremely small. The highest beach, rising northward from 1134 feet at Duluth to 1207 feet on Mt. Josephine, (supposing Lawson's observations at the latter place to be correct) must meet the beach which passes northward from Sault Ste. Marie somewhere in the north, because they are the margin of one water plane. It seems almost certain, therefore, that the highest beach must rise more rapidly northward from Sault Ste. Marie than it does from Duluth. For it starts from a level about 120 feet lower. I am bound to conclude, therefore, for the present that the highest shore line rises at least 1½ to 2 feet per mile northward from Sault Ste. Marie and keeps well up along the north shore, leaving the open straits to the north, as stated in my paper. (As I remember Prof. Lawson's figures, the Missinaibi pass is about 1044 feet above sea level and the Kenogami pass 1109 feet.) I see no escape from this conclusion, then, unless it be by some abrupt and very exceptional dip of the ancient water plane downward to the north. It would appear, therefore, that the mean height of the highest shore line in the Superior basin can hardly be less than 1100 feet or more than 1200. On present data the most probable mean that I can name for the whole basin is about 1150 feet. I think this estimate is conservative and likely to prove too low rather than too high.

Second. As to the origin of the highest beach, I believe that it was all made at sea level and that the Iroquois beach in the Ontario basin, the highest beach in the northern parts of the Huron and Michigan basins with that of the Superior basin, and also the upper beach of the Red river and Winnipeg basin, are all one continuous beach and of the same age in all parts. In the Michigan and Huron basins this beach slopes downward toward the south and passes under the present lake level, thus leaving Lake Erie with all its beaches and also the Chicago outlet entirely above that plane. The Erie beaches are undoubtedly the shores of an ice-dammed lake of the glacial recession. It was much the largest lake produced in this way. But there were other smaller, shorter lived ones at the south ends or sides of all the basins. The Ontario basin was probably also partly filled by a large temporary lake of this kind, emptying through the Mohawk valley. But the record it may have left has been wiped out by the later marine invasion which made the Iroquois beach.

Third. As to the relations of Niagara, Lake Erie, etc., I have the data to show conclusively that the present period of the great cataract's activity began at a point a few rods above the cantilever railroad bridge. Before that time the outlet of the upper lakes was eastward over Nipissing pass and down the Ottawa river, and greater Niagara was replaced for a long period of time by a very much smaller stream (the "Erigan" river), which drained only the Erie basin and possibly not all of that. This smaller stream made the narrower, more shallow gorge of the whirlpool rapids, extending from Foster's flat below the whirlpool up to the cantilever bridge. The time which was required to do this must have been many times longer than Mr. Gilbert's estimate of the age of the whole gorge from Lewiston up. I should say that the multiplier would have to be tens rather than units. As a measure of the duration of postglacial time, therefore, I do not see how the gorge can have any value worth mentioning. But without meeting this demand, it goes far enough to show that postglacial time must have been much longer than the current estimate.

But to go a step farther, there is considerable evidence to show that the greater Niagara had another earlier period of postglacial activity, during which it cut out the gorge from Lewiston to Foster's flat. Overhanging Foster's flat is the old fall ledge of the greater cataract with the narrower gorge of the smaller stream cut back on the south side. At this old ledge the great cataract ceased for a long period of time, and when it returned the smaller stream had cut back in the interval to the cantilever bridge. Thus the great cataract has been intermittent in its activity. It has had two active periods separated by the long, almost indefinitely long, period during which the falls of the smaller stream carried on the work. And all this *plus* a short period for the Chicago outlet has been since the ice age. There

is some evidence in the beaches of the north pointing to this earlier period of the cataract. But this episode of the gorge is not yet so strongly established as the other two.

Fourth. A word as to the nature of the changes that have produced these great alterations of land altitude. They are so extensive, so great, and above all so *recent* as to be almost appalling when one thinks of them.

Of the whole amount of apparent elevation along the line of the parallel of 46° 30′, which passes close to Duluth, Sault Ste. Marie and North Bay, I think I can make out fairly well that between 600 and 800 feet of it are due to a change of the *secular* kind, while the rest is of the *paroxysmal* sort, I have always embodied this distinction (which is derived from Dana's Manual of Geology) in my working hypothesis, and with good effect. I am quite sure that it was the secular factor that changed the outlet of the upper lakes this last time and brought Niagara into its present state of activity—in consequence of a secular relative, differential, northward elevation. It seems probable that the earlier episode of Niagara was also due to the same factor. But the order of relative change was probably then in the opposite direction. I believe that maximum northern relative elevation coincided with maximum glaciation, as you have said. But I believe, further, that maximum relative northern submergence coincided with the climax of the postglacial warm epoch now passing off. Relative northward elevation has been in progress for some time past and we are now advancing again toward a climax of cold."

3. *Introduction to Geology as historical science.* Einleitung in die Geologie als historische Wissenschaft. Beobachtungen über die Bildung der Gesteine und ihren organischen Einschlüsse. I Theil: Bionomie des Meeres. II Theil: Die Lebensweise der Meeresthiere. III Theil: Lithogenesis der Gegenwart (by Johannes Walther), pp. i-xxx and 1055. (Gustav Fischer), Jena, 1893–1894.—The author describes the method employed in this "Introduction to Geology" as the *ontological method* of investigation, which consists in the interpretation of the events of the past by the phenomena of the present. The aim he has held before him while constructing this elaborate work has been to bring together from many sources, from periodicals, monographs and special treatises, all the facts bearing upon the general question of the formation of rocks and upon the conditions under which organisms have lived and died and been fossilized.

The work is divided into three parts: The first (Bionomie des Meeres), on the marine conditions of life, was briefly noted when it first appeared in 1893.* It is an elaborate description and classification of the zones of life-habitat of the ocean following the system of classification proposed by Haeckel† and gives a very convenient and apparently exhaustive synopsis of the facts with full references to sources.

* This Journal, III, vol. xlvi, p. 240.
† Planktonstudien, Jena, 1890.

The second part (Die Lebensweise der Meeresthiere, pp. 197–532) treats of the mode of life of sea animals. In its 335 pages are brought together the facts known about the geographical distribution, bathymetric range and other conditions of adjustment to conditions of life environment of the organisms whose remains could be preserved as fossils. The facts are classified in sections for each of the larger groups of organisms, the classes or branches. Here also references to original sources are liberally given.

The third part (Lithogenesis der Gegenwart, pp. 535–1055) describes the methods of formation of rocks on the present surface of the earth. The first section is devoted to the statistics of Lithogeny or dynamical geology, classified under the heads of Weathering, Ablation, Transportation, Corrosion, Deposition, Diagenesis and Metamorphism. The second section describes and classifies the various areas of the present surface of the earth, (Du Faciesbezirke der Gegenwart) distinguished by differences of climate or physical surface as affecting the geological formations or phenomena. In the last section the outlines of a comparative Lithology are discussed under such headings as the correlation of facies, the equivalency of rocks, changes of facies.

Although there is little original in the book, a large amount of statistics scattered over a very wide range of literature are brought together, concisely stated and classified in such a way as to be helpful and suggestive to the geological student, to which end the full references to original sources and the good indexes at the close contribute no small share. H. S. W.

4. *Alabama, Geological Survey—Geological Map of Alabama with explanatory Chart.* E. A. Smith, State Geologist and Assistant. Two large folio sheets. 1894.—The map presents in clear and yet not too strong colors the geological features of the State, with division of the scale as fine as the classification in the reports. The Chart presents an admirable synopsis of the important features of the geological conditions of the State. In the first column names, synonyms, classification and common fossils are given for each of the recognized formations, opposite each division in the second column are given thickness, lithological and topographical characters, area and distribution, in a third column, useful products followed by a fourth describing the soils, characteristic timber growth and agricultural features, and a final column in which are listed the Reports in which each of the formations is described, making a convenient index to the geology of the State for the general reader as well as the geologist.

5. *Manual of Geology;* by James D. Dana.—The new (fourth) edition of this work, entirely rewritten and much enlarged, will be ready about the middle of January.

6. *Bulletin of the Department of Geology, University of California.* A. C. Lawson, Editor, Berkeley, Cal.—This work has been noticed in this Journal, vol. xlvii; p. 147, 1894, and we are now in receipt of Nos. 5, 6 and 7.

No. 5. *The Lherzolite-Serpentine and associated rocks of the*

Potrero San Francisco; by CHAS. PALACHE.—This presents the results of a study of this rock both in the field and in the laboratory. The facts obtained show that it is the result of alteration of a very basic igneous rock intruded into the San Francisco sandstone.

No. 6. *On a rock from the vicinity of San Francisco containing a new Soda-Amphibole;* by the same author.—The rock consists of fine grains of albite. Through this is thickly sprinkled the amphibole in small acicular crystals, in simple forms. The optical relations determined in thin section are $b = \mathfrak{b}$, $c : \mathfrak{a} = 11° - 13°$ in front: Pleochroism strong: $\mathfrak{a}$=sky-blue to dark blue, $\mathfrak{b}$=violet, $\mathfrak{c}$=yellowish brown to greenish yellow. Absorption $\mathfrak{a} \geqq \mathfrak{b} > \mathfrak{c}$. Is often zonally built. A chemical analysis gave the following results:

SiO_2	Al_2O_3	Fe_2O_3	FeO	MnO	MgO	CaO	Na_2O	K_2O	H_2O	Sum.
55·02	4·75	10·91	9·46	tr.	9·30	2·38	7·62	0·27	? =	99·70

A consideration of the molecular ratios of the oxides from this analysis shows them to be

SiO_2	·918		FeO	·131		Na_2O	·123		
Al_2O_3	·046	·114	MgO	·233	·407	K_2O	·003	·126
Fe_2O_3	·068		CaO	·043				

whence $(Na, K)_2O : R_2O_3 : SiO_2 :: .126 : .114 : 480 :: 1·05 : 0·95 : 4$
and $RO : SiO_2 :: ·407 : ·437 :: 1 : 1$.

From which it appears that the mineral is a mixture of the actinolite $(Ca, \underset{III}{Mg, Fe})SiO_3$, the glaucophane $NaAl(SiO_3)_2$ and the riebeckite $NaFe(SiO_3)_2$ molecules. The author proposes for it the name *crossite* after Mr. Whitman Cross of Washington.

The propriety of giving new names to indefinite mixtures of isomorphous molecules is at best a very doubtful one, unless from constancy and frequence of occurrence there is a positive demand for such a term. By far the larger part of the work of chemical mineralogy in the last few decades has been ridding the terminology of just such synonyms; and one needs only to glance at the index to the recent edition of Dana's Mineralogy to become convinced of this fact. The best usage seems to require that definite mineral molecules once established should receive definite names, and isomorphous mixtures should be classed under the one prevailing.

No. 7. *Geology of Angel Island;* by F. L. RANSOME.—This is a very careful and excellent study of the geology and petrography of a small island in San Francisco bay. The island consists in the main of the San Francisco sandstone which is folded, and intruded with serpentine and a basic igneous rock classed by the author under fourchite. It is accompanied by a detailed geological map and plates, and DR. G. H. HINDE adds a chapter on the idiolarian chert found on the island. L. V. P.

7. *Mineral Resources of the United States.* Vol. IX. Calendar Year, 1892, 850 pp. Washington, 1893. Vol. X. Calendar Year, 1893, 810 pp. Washington, 1894. DAVID T. DAY, Chief of Division of Mining Statistics and Technology (U. S. Geol.

Survey, J. W. Powell, Director).—The latest volumes of this valuable series are Nos. IX and X for the years 1892 and 1893 respectively. They have been issued under the able editorship of Mr. David T. Day, and like the earlier volumes, contain a large amount of interesting and useful data in regard to the present condition and recent growth of the mineral industries of this country.

8. *Physikalische Krystallographie und Einleitung in die krystallographische Kenntniss der wichtigeren Substanzen;* von PAUL GROTH. Dritte vollständig neu bearbeitete Auflage, I und II Abtheilung, physikalische und geometrische Eigenschaften der Krystalle, 528 pp. 8vo. Leipzig, 1894 (Wm. Engelmann).—Ever since the publication of the first edition in 1876, the *Physikalische Krystallographie* of Professor Groth has been the standard work in this subject for the mineralogists of all lands. In the second edition, issued in 1885, the work was rewritten throughout and much enlarged, and the same is true to even greater extent of the present third edition. The author's keen mind and long experience as a teacher have enabled him to present the subjects of crystallography and the general physics of crystals with rare clearness and system, as all know who have used the earlier editions. This is conspicuously true of the work in its present form, which contains much that is new and suggestive particularly in the discussion of the symmetry of crystals and their classification based upon this. Two parts of the third edition only have thus far appeared, but the third, upon the methods of calculation applied to crystals, instruments and methods of investigation, is promised in a few months.

9. *A Manual of Microchemical Analysis;* by Professor H. BEHRENS, of the Polytechnic School in Delft, Holland, with an introductory chapter by Professor JOHN W. JUDD of the Royal College of Science, London. 246 pp. 8vo. London and New York, 1894 (Macmillan & Co.)—Workers in Petrography and all who have occasion to use the now well developed microchemical methods will welcome this admirable volume. The original contributions of the author to this subject are well known, but his work on *Mikrochemische Methoden* is not readily accessible to the English student and hence this translation made by him will be highly appreciated. The value of the work is increased by the fact that the author's manuscript has been revised by Prof. J. W. Judd, who with the assistance of Mr. A. E. Tutton, has also seen it through the press. A brief but interesting introduction, in part historical, in part descriptive, has been added by Professor Judd.

10. *Handbuch der Mineralogie;* von Dr. C. HINTZE. Achte Lieferung, pp. 1121–1280. Leipzig, 1894 (Veit & Co.).—The eighth part of Hintze's great work has appeared recently. It embraces the close of the Pyroxene group, the Amphibole group and the opening pages on the species beryl.

III. Botany.

1. *Lehrbuch der Botanik.* Dr. Frank, Berlin, 8vo, two volumes, pp. 669, and 431 (1892 and 1893).—Professor Frank's text-book follows the usual course of treatment, but gives great prominence to the subject of physiology. This part of his work makes the treatise of great value. The topics are fully treated and the essential matters are kept in their proper relations, so that the general result is symmetrical. The questions with which Professor Frank has specially busied himself in his original work, are discussed in an exceedingly interesting manner and with due perspective. In a few instances, it seems on the first reading, as if the author had stated some of his own conclusions in too positive a form for the requirements of a text-book for the general student, but, as above said, the treatise is symmetrical. G. L. G.

2. *Lehrbuch der Botanik.* Dr. K. Giesenhagen, Munich, 8vo, pp. 335, 1894.—The author has prepared this text-book with reference to the needs of students who are reviewing their work before examination. It is, therefore, a comprehensive volume in which the whole ground is covered in such a manner as to refresh one's memory with regard to the results of personal laboratory exercises in histology and physiology, supplementing this by sufficiently full treatment of systematic and economic botany. The newer results have been incorporated with the older in a well-proportioned and well-balanced manner, so that the work gives a clear and sound exposition of the present state of our knowledge. The author has carried his work on under the advice and with the assistance of Dr. Goebel, Professor of Botany in Munich. G. L. G.

3. *Lehrbuch der Botanik.* Dr. F. Pax, Leipsic, 8vo, pp. 365. —This is the ninth edition of Prantl's well-known work, already noticed in this Journal. The revision has been thorough. In the histological and physiological portions changes demanded by recent investigations have been made, and important modifications have been made throughout the systematic part. It may be remembered that the earlier editions gave a good deal of prominence to species: in this edition, only those are referred to which are of interest in economic botany, especially in medicine. The number of illustrations has been increased from 326 to 355. The improvement in the engravings since the first edition has been very great. G. L. G.

4. *Lehrbuch der Botanik.* Dr. E. Strasburger, Dr. F. Noll, Dr. H. Schenk, Dr. A. F. W. Schimper, Jena, 8vo, pp. 558, 1894.—Professors Strasburger and Schimper with the two Privatdocents associated with them in botanical teaching in Bonn, have carried successfully to completion the very hazardous experiment of preparing a composite text-book. The dangers which confront such an undertaking are obvious. Each specialist is likely as we say to magnify his office, and give a disproportionate amount of space to the results which have been recently attained

in his part of the field. It is exceedingly difficult to secure under such circumstances anything which approaches careful editing. Distortions of a serious character are very likely to result. Furthermore it is very hard to avoid duplication, even when the conferences and friendly discussions have been frequent and critical. But the outcome of this experiment has been successful in a high degree. Professor Strasburger has given the subject of histology the fruits of his long and ripe experience in investigation and teaching. Within the comparatively narrow limits of 130 pages, he has compressed without too much condensation, all the essential facts of general and special anatomy of the external parts and the internal structure. It is only when one looks over this part of the volume a second time that he begins to realize how much grain free from chaff has been made ready for the student. Moreover all the material has been arranged in an orderly and attractive manner. Dr. Noll has considered the subject of physiology in a comprehensive fashion, dealing with the principal phenomena presented by all the organs of flowering and flowerless plants. He has presented his facts fully, but without prolixity. The style is clear, and the illustrations like all which are given in the volume, are of a high order considered both from a scientific and a pedagogic point of view. We think he has done well to give so much prominence to the experimental side of the subject. Dr. Schenck and Professor Schimper divide the field of systematic botany between them, the former taking the Cryptogams, and the latter the flowering plants. It is enough to say that the work is satisfactory in every respect. New and admirable figures, many of them of the highest excellence, illustrate the remarkably clear text. As might be expected, the subject of adaptive modifications, although touched but lightly, has assumed a peculiar charm at the hands of Professor Schimper. A good deal of new light is thrown on the subject and the whole of it is invested with a deep interest. Returning for a moment to the matter of illustrations, attention must be called to the beauty and accuracy of the colored figures which represent poisonous plants. It is to be hoped that a translation into English of this admirable work will soon be in the hands of English-speaking students.

G. L. G.

5. *A Student's Text-Book of Botany;* by SYDNEY H. VINES, M.A., Professor in the University of Oxford. First Half, 8vo, pp. 480. London, 1894.—The author has based his work on the well-known *Lehrbuch der Botanik*, of PRANTL, which, as will be seen in another notice, has reached its ninth edition in Germany. But the changes which he has made, are so numerous as to transform this into a new treatise; the metamorphosis is almost complete. A great deal of new matter designed for the advanced student has been skillfully interwoven, bringing the whole well up to the most advanced knowledge of minute details. It is to be remembered that there are scores of trained workers now engaged in minute investigations in different parts of the field

covered by this volume, and that the results of their investigations are swelling the periodical literature of the science at a startling rate. It is indeed high praise to confess that a symmetrical résumé like this is fairly abreast of the times.

One feels inclined to criticise the free use of the new terminology adopted and in part suggested by Professor Vines, although there can be little doubt of the utility of the introduction in an advanced work of this sort of the new terms which are employed so generally in monographs; the work serves as an excellent technical dictionary. But we own that we should have been glad to see from Professor Vines, whose scholarship would have carried great weight, a reform in the terminology. Our language has, of course, lost its plasticity, and few new terms can be constructed out of English stems and roots; we are driven back to Greek for our materials and these serve every purpose; but it seems as if the new terms demanded by the advance of knowledge could be framed with some regard to euphony. Perhaps no one in English-speaking countries is so well prepared as Professor Vines to undertake this task, and perhaps he will take it in hand when he thinks the time is ripe. G. L. G.

6. *Practical Physiology of Plants;* by FRANCIS DARWIN, F.R.S. and E. HAMILTON ACTON, M.A., Cambridge, 1894, 8vo, pp. 321.—This is an outline of directions for experimenting. Explicit directions are given for conducting, generally with simple apparatus, the more conclusive experiments in nearly all parts of the field. As is quite proper, the character of the results are seldom announced; the student must find out for himself. In Detmer's Practicum and in the work of Oels, the student is generally helped towards his result by a brief statement of what he may expect to discover if he is successful, and this is a good plan for a certain class of students. But there is no question that the method used by Mr. Darwin and Mr. Acton is pedagogically correct, and is capable of giving excellent results. The only drawback to physiological experimenting at the hands of students, who are simultaneously pursuing other subjects, is the great amount of time which is generally consumed while plants are growing or reacting, and the consequent difficulty of arranging hours so as to make a close economy of time. The student should receive some hints as to what he might be getting ready for the next study while he is patiently waiting for something to happen to the plants which he has in hand, just as in the chemical and physical laboratory he is taught to keep many things going at the same time. It seems, on first reading, that the authors have arranged the work admirably in order to secure the greatest economy of every moment. This we are putting to a practical test.

G. L. G.

7. *A practical Flora for Schools and Colleges;* by O. R. WILLIS, New York, 8vo, pp. 349, 1894.—After a very brief statement of the subdivision of the subject of Botany, Professor Willis begins with an analytical key to the natural orders, and thence

passes to a description of certain orders. It is not easy to see the principle which guided him in the choice of subjects; in fact, it seems as if hardly any principle at all had been followed, but leaving this aside, it must be stated that a great amount of useful material has been brought together, and in such form as to be readily utilizable by a teacher. The work is a convenient handbook of Economic Botany. There are some unfortunate omissions, but, on the whole, there is a larger mass of well-arranged facts made ready for the hand of the teacher than we remember to have seen in any English treatise on the subject. The name of the valuable work seems to us a misnomer. Certainly it does not suggest to instructors that in these pages they can find the information regarding useful plants, in search of which they range through cyclopedias of every sort. G. L. G.

8. *Pflanzen-Teratologie, systematisch geordnet.* Dr. O. PENZIG, Genoa, 1890, and 1894.—The first volume of Professor Penzig's masterly work, comprising the polypetalous dicotyledons, was published four years ago; the concluding volume, in which are considered all the other groups of plants in which monstrosities have been thus far detected, was finished in June of this year, and has just been issued. The minuteness of Professor Penzig's search for recorded cases of monstrosities is shown by references to some of the most obscure sources, such as local journals and the like, and his care in stating the appearances of the distortions has been such as to place in the hands of the reader exact and yet much abbreviated descriptions which can be safely used in generalizations. On the part of the author there has been no attempt to state theoretical views in connection with the special cases, except where such treatment appears absolutely necessary; but he gives, in a clearly written preface to the second volume, a sound and clear exposition of modern speculations in regard to this interesting subject. With this treatise and with the philosophical work of Dr. Masters, the student of this subject is well-equipped. G. L. G.

9. *Practical Botany for Beginners;* by F. O. BOWER, F.R.S., Professor of Botany in the University of Glasgow, London, 1894, 8vo, pp. 275.—With the caution given in the preface, this laboratory manual, an abridgement of the larger Course of Practical Instruction in Botany, can be recommended without reserve. The caution is worth heeding in many quarters. "Type-teaching in Biological Sciences appears at present to be inevitable in elementary classes; it lies chiefly with the teacher to avoid the evils which are apt to arise from it. In order to use this book with proper effect, his knowledge should extend far beyond the area of the work here specifically described, and the larger edition may help him towards this end. By grasping every opportunity of comparison of the type selected with allied forms which show differences of detail, he will then be able to guide the pupil to distinguish essentials from secondary details, and to check the dangerous tendency of beginners towards generalization from too limited an area of fact." G. L. G.

IV. Miscellaneous Scientific Intelligence.

1. *New Tables of the Planets.*—Prof. Newcomb in his report as Superintendent of the Nautical Almanac announces the substantial completion of the work of determining the fundamental constants of astronomy and the elements and masses of the major planets from Mercury to Saturn inclusive. He hopes to have the tables from Mercury to Mars inclusive completed by the end of the fiscal year. The tables of Jupiter and Saturn were completed by Dr. Hill during the last fiscal year. This leaves only Uranus and Neptune to be worked up.

Prof. Newcomb takes this occasion to present a general report upon the nature and objects of this work. The best of existing tables, those of Leverrier are by the lapse of time and by the lack of homogeneity in the elements employed, quite unsuited to the present needs of astronomers. There was needed a partial or complete reduction of all good observations of the sun and planets made since 1750. The actual number of separate meridian observations of the sun and the three planets Mercury, Venus, and Mars used by Prof. Newcomb was 62,030, against 10,893, used by Leverrier. The work of computation required to utilize all these observations and determine simultaneously the 23 unknown quantities involved can be appreciated only by those who have experience in such work. This immense labor has been secured by means of extra appropriations of $3,000 to $4,000 annually, together with such economy in the regular work of the Almanac office as was practicable.

Prof. Newcomb speaks of the increased demand for accurate positions of a much larger number of fixed stars than have heretofore been given in the Ephemeris. This increase is greatly needed, and in making it, we hope that the additional stars will be selected after consultation with the superintendents of the British, German and French Almanacs. A common list of stars may or may not be desirable. If a common list is not desirable, the differences in the lists should be for explicit reasons.

2. *Ostwald's Klassiker der exakten Wissenschaften.* Leipzig (Wm. Engelmann).—This series of classical scientific memoirs continues to grow in value as it grows in size. The papers, selected for republication, are well chosen and the originals being for the most part difficult of access, the library becomes of the first importance to the student in the subjects which it embraces. The latest additions are as follows:

No. 54. Anmerkungen und Zusätze zur Entwerfung der Land und Himmelscharten von J. H. Lambert (1772) 93 pp.

55. Ueber Kartenprojection: Abhandlungen von Lagrange (1779) und Gauss (1822) 101 pp.

56. Die Gesetze der Ueberkaltung und Gefrierpunktserniedrigung. Zwei Abhandlungen von Sir Charles Blagden (1788). 49 pp.

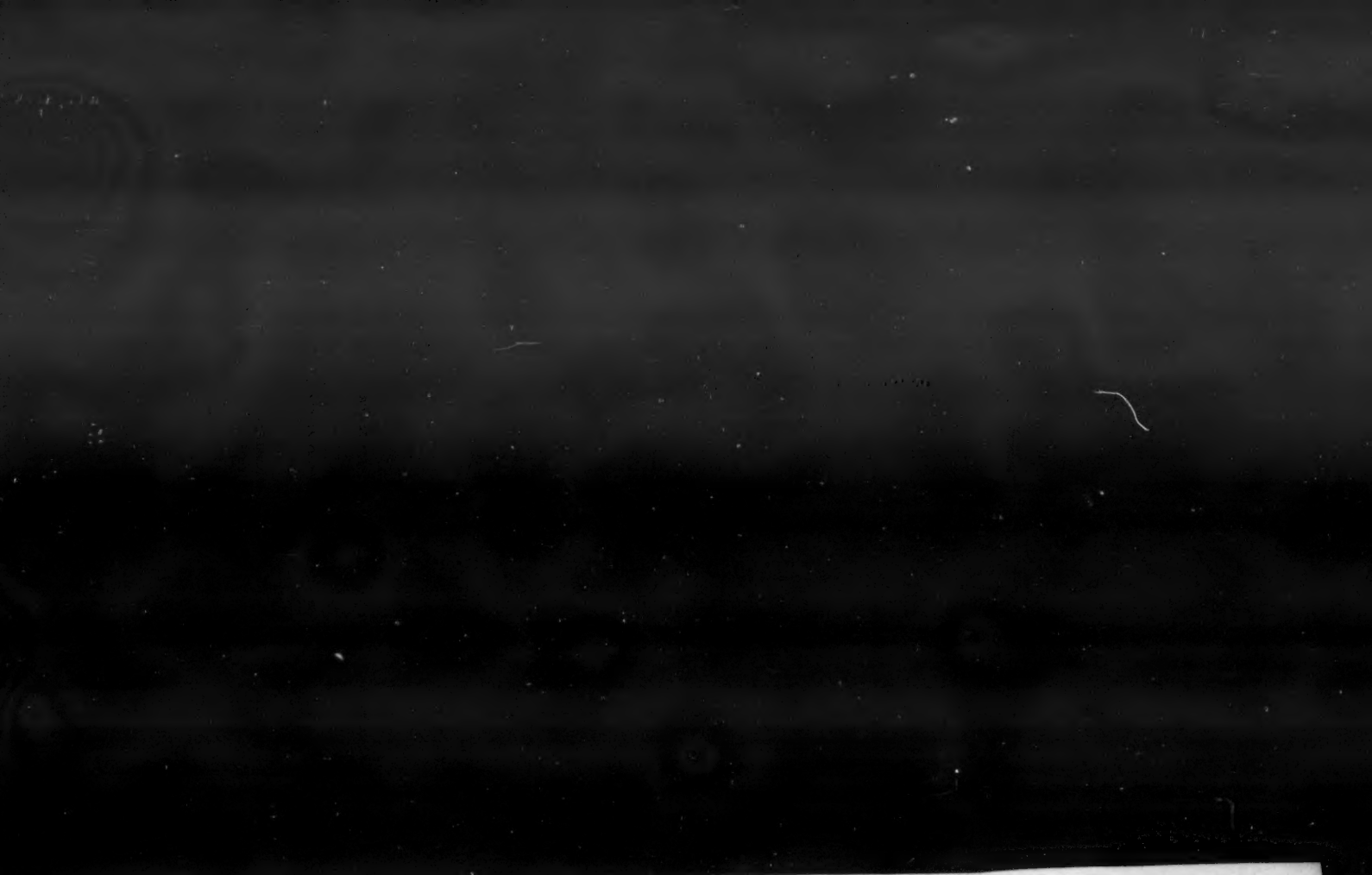
57. Abhandlungen über Thermometrie von Fahrenheit, Réaumur, Celsius (1724, 1730–33, 1742). 140 pp.

58. Chemische Abhandlung von der Luft und dem Feuer von Carl Wilhelm Scheele (1777) 112 pp.

59. Otto von Guericke's neue "Magdeburgische" Versuche über den leeren Raum (1672) 116 pp.

Obituary.

Lewis R. Gibbes.—Dr. Lewis R. Gibbes, a scientist of very varied learning, died on the 21st of last November at Charleston, S. C., in the 85th year of his age. He was born in Charleston on the 14th of August, 1810. He entered on a course of medical study in 1830; but before the close of 1831 he was appointed Tutor in Mathematics in the College of South Carolina, at Charleston. At the revolution in the College, of December, 1834, when, owing to opposition to the President, Dr. Cooper, all the officers were requested to resign, he was set adrift with the rest. But in the new organization which followed the next day, Mr. Gibbes was made Professor of Mathematics. He resigned his position the following year, and in 1836 visited Paris for the purpose of completing his medical education, and gratifying his desire for knowledge in other directions. In 1838 he was again a Professor in the Charleston College, teaching mathematics, physics, chemistry and mineralogy. Botany and some departments of Zoology were also among his special studies. He published various papers on the Crustacea of the American coast, and showed his comprehensive knowledge of the subject by his revision, in 1850, of the species in the various United States collections, to which he added valuable notes and descriptions of new species. His encyclopedic tastes and knowledge are further shown by his astronomical work, especially between 1845 and 1854, when he published, besides other related papers, one on a comparison and discussion of all the observations made in the United States on the transit of mercury of May, 1845, and in 1849–50, while engaged in observing occultations of fixed stars by the moon, devised and constructed an occultator for the purpose "of obtaining the approximate times of disappearance and reappearance with less labor than by calculation."

As President Shepherd in his tribute to Dr. Gibbes, says: Astronomy was his passion, but he cultivated nearly every phase of our complex modern science, and cultivated none that he did not dignify and adorn. As a teacher he was lucidity itself. He not only taught so that he might be understood by the pupil, but so that he could not be misunderstood. He ever manifested a spirit of absolute consecration to his ideal as a scientist, an insatiable quest of knowledge, undiminished even in the view of death, and respect and reverence for the eternal verities of the Christian faith.

The death of Father Denza, Director of the Vatican Observatory, is announced by telegraph.